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ON THE CLASSICAL THEORY OF COMPETITION, VALUE AND PRICES OF PRODUCTION*

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Introduction

In many interpretations of the classics it has been the practice to describe the properties of their theories in terms of the general equilibrium theory (see Arrow and Hahn, 1971, ch. 1; Stigler, 1957; Samuelson, 1978; Hollander, 1973, 1980). However, in more recent discussions doubts have been raised as to whether the theory of the "invisible hand" of Smith, and the theory of "free competition" in Ricardo and Marx, can be interpreted in the framework of general competitive analysis as, for example, presented by Arrow and Hahn (1971). On the other hand, there have been attempts made to utilise the classical framework for economic analysis and to extend it to the theory of imperfect competition (see Koshimura, 1975, 1978; Okishio, 1956; Teplitz, 1977; Nikaido, 1975). Yet the special character of the theory of competition in the classics and the supply and demand mechanism in the classics has not become a topic of discussion until recently (see Garegnani, 1983; Benetti, 1981; Cartelier, 1981 Deleplace, 1981). In most of those articles, it is suggested that Smith, Ricardo and Marx did not have an equilibrium concept of price but rather a concept of a centre of gravitation. It is maintained that they had a concept of two different laws for price determination: one determining the long-run production price, the other one determining the market price, *i.e.* the fluctuation of the actual price. The first part of the paper elaborates the difference between neoclassical general competitive analysis and the classical theory of competition, and demand and supply analysis. In doing this, the role of demand for relative prices is explored in the classics and, in addition, Marx's dynamic theory of competition is compared with the one of the classics and the neoclassics. Whereas the first part of the paper mainly summarises results recently elaborated, on the stability or instability of the classical mechanism of the competitive process, the second part of the paper analyses some new problems related to the change of the centres of gravitation. Here the question will be pursued, as to how robust these centres of gravitation are when small changes in the structure of production or income distribution occur. By applying sensitivity and error analysis to linear production models, small and strong shifts in the centres of gravitation will be studied. In this context, the problem will be discussed as to whether in cases of strong shifts of the centres of gravitation, the market mechanism would allow for such

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changes. The appendix provides some proofs of the statements made in part two of the paper.

I. On Theories of Competition

1. *On Neoclassical Theory: Competition and Convergency Toward Equilibrium*

Neoclassical general competitive analysis usually interprets the classics as forerunners of a theory of perfect competition (see Arrow and Hahn, 1971, ch. 1, and Stigler, 1957). But this — as many writers have pointed out recently — does not seem to be correct. There are of course, some elements in the classical theory particularly in Smith's work, that lend themselves to the neoclassical point of view. Competition, in Smith's sense, meant that everyone should be able to act according to their self-interest. There should be as few barriers as possible to economic activities and the pursuit of self-interest would maximise the welfare of the society as a whole. The market was seen to be the place where the individuals and their interests would be co-ordinated and disturbances eliminated. The mechanism which would produce these results is the supply and demand mechanism operating in a practical milieu of "perfect liberty". This is at least the standard neoclassical interpretation of Smith's theory of the invisible hand and in that respect, Smith was seen as the progenitor of the neoclassical theory of competition (Arrow and Hahn, 1971, p. 2).

However, the question must be raised as to whether or not the "invisible hand" in Smith and "free competition" in Ricardo and Marx meant the same as the competitive forces in neoclassical equilibrium analysis. The neoclassical theory formulated several conditions under which a competitive equilibrium with the properties of a Pareto-optimum would exist. These conditions, generally regarded as the main conditions for a perfectly working competitive market system, are the following: there are convex production and consumption sets, the agents of the economic activities (producers and consumers) exhibit maximising behaviour, and their decisions are made independently of each other. There is no co-operative behaviour or collusion and there are no external effects. Moreover, there are an "infinite number" of economic agents (Kirman, 1981, p. 160) such that there is no influence on the quantities sold or bought (price takers) and there must be a perfect mobility of resources and complete information. Given these preconditions, together with the initial endowments of resources and consumer preferences, the competitive process among producers and consumers brings about equilibrium prices which allow for a consistent exchange of commodities between the participants in the markets. Moreover, the market mechanism will provide, not only consistent exchange ratios (existence of at least one equilibrium), an optimal allocation of resources (Pareto-optimaility)[1] and a fair income distribution, but the market mechanism will also entail the elimination of perturbations (stability, at least under the condition of gross substitution). The

[1]There are also more general neoclassical economic models which try to show that a Pareto-optimality can be achieved even if the number of firms decreases (oligopolistic or monopolistic competition). But in the case of a small number of firms in an industry, increasing returns to scale, entry and exit barriers for the markets and the presence of uncertainty and risk, it is quite difficult to derive a unique equilibrium with a welfare optimum. See Arrow and Hahn (1971), Negishi (1961), Nikaido (1975).

fundamental mechanism which brings about these results is the supply and demand mechanism (Arrow and Hahn, 1971, p. 265). Supply and demand are dependent on market prices and market prices respond to excess demand or supply in each market. This adjustment process is conceptualised as a process of convergence of prices and quantities toward equilibrium prices and quantities. The condition for convergence is usually demonstated as follows. By referring to discrete price changes a price mechanism can be called stable: if

$$\| f(p_t) - f(p^0) \| \leq \mu \| p_t - p^0 \| .$$

Where $f(p_t) = p_t + M(p_t)$, $M(p_t)$ an adjustment function given, for example, by $k\,z(p)$, with $z(p) = D(p) - S(p)$ the excess demand function, $0 < k < 1$, $0 < \mu < 1$, and $\| \quad \|$ the Euclidean vector norm. The inequality above means that a perturbation of prices in the neighbourhood of p^0 will vanish, when $t \to \infty$ and actual prices p_t converge toward the equilibrium price vector p^0 with $z(p^0) = 0$. However, instead of characterising stable mechanisms by their response to price perturbations, neoclassical writers can equivalently study their response to the excess demand functions (Smale, 1981; Hahn, 1982 and Jordan, 1983). Competitive forces are assumed to be equilibrating forces and both the equilibrium prices and the convergence of actual prices toward equilibrium prices are determined by the supply and demand mechanism. A change of parameters, such as a change in techniques, the structure of demand, or initial endowments, will in a "regular economy" (Dierker, 1982, p. 795) -- the market agents passively responding to it -- lead to a new nearby competitive equilibrium. Thus an equilibrium will not be brought about by a discontinuous process and disruptions, but is a result of a continuous smooth process of convergence, during which disequilibria will vanish. Of course, as demonstrated recently, under conditions of uncertainty, money, exchange at disequilibrium prices and strong quantity adjustments equilibria might not exist or market mechanisms might not be stable for prices as well as for quantities (see Arrow and Hahn, 1971, ch. 16; Hahn, 1982; Dierker, 1982; Varian, 1977).

However, the above outlined neoclassical competitive analysis is usually taken as the neoclassical standard model of competition according to which the classical economists are interpreted (Arrow and Hahn, 1971, ch. 1; Samuelson, 1978; Hollander, 1973, 1980).

2. *On the Classical Theory: Centre of Gravitation and Fluctuations of Market Prices*

Classical political economy has developed a notion of the competitive process and of supply and demand which seems to differ from the ones in neoclassical general competitive analysis. Many writers on the classics such as Garegnani (1976, 1981, 1983), Roncaglia (1978), Bharadway (1983), Benetti (1981), Cartelier (1981) and Deleplace (1981) have shown that the main features of classical political economy are the concept of economic surplus, the concept of centre of gravitation and the particular role of supply and demand. These three are essentially related to the concept of competition in the classical scheme.[2]

[2]The following interpretation of the classics follows more the neo-Ricardian view. The difference between the classics and Marx will be discussed in the next section. One of the main assumptions here is the given physical system.

(1) The classical political economy assumed that, once the technical conditions of production (matrix A), the real wage vector (d) and the vector of direct labour requirements (l) are given, the system of production generates a surplus product (S) that can be distributed among the remaining classes of the society. Since, in the classical theory, workers' consumption is regarded as a necessary part of the social reproduction, the surplus is defined as:

> social product -- replacement of means of production — necessary consumption = $X - (A + dl)X = S$ (surplus product)

This system of production is assumed to be a productive one. The exchange-values of the reproducible commodities, according to classical political economy, are determined by their cost of reproduction. The costs of reproduction of the commodities are considered as the centre of gravitation for the market prices, the actual prices.

(2) In Adam Smith, the natural prices are considered to be the centre of gravitation.[3] The natural prices are composed of the normal rewards of the factors of production (wage, profit, rent). For Ricardo, and later Marx, in a first approximation, the direct and indirect labour requirements are regarded as the centre of gravitation for actual prices. The natural prices for the commodities and the natural prices for the factors of production in Smith's sense are independent of short-run demand and supply (see Bharadway, 1983).

The natural prices are determined by their long-run components. These components, however, are regarded as independently determined from the supply and demand mechanism. Yet, it is assumed that there is a tendency to equalise the rates of return on the factors used up in production, enforced by the possibility of the factors to move from areas of low to high returns. Nonetheless, also the tendency toward differential profitability and differential wages was studied by the classics (see Semmler, 1984, ch. 2).

If we assume equalised rewards for the factors of production and do not consider rent, *i.e.*, the price of land, then according to Pasinetti (1973) we may write the natural prices — *i.e.*, the centres of gravitation in Smith's concept — as vertically integrated wages and profits:

$$p = wl + pA + rpB$$

$$p(I - A) = wl + rpB$$

$$p = wl(I - A)^{-1} + rpB(I - A)^{-1}$$

[3]Smith speaks of such centres of gravitation when he develops the notion of natural price.
"The natural price, therefore, is, as it were, the central price, to which the prices of all commodities are continually gravitating. Different accidents may sometimes keep them suspended a good deal above it, and sometimes force them down somewhat below it. But whatever may be the obstacles which hinder them from setting in this centre of repose and continuance they are constantly tending toward it." (Smith, 1961, p. 65).
Ricardo has a similar concept concerning values (Ricardo, 1951, p. 91).

the capital stock matrix, p the price vector, r the uniform profit rate, w the wage, and l the vector of direct labour requirements per unit of output. Thus, we can write the price for a commodity:

$$p_i = w_i' + \pi_i' \quad .$$

w_i' and π_i' are the vertically integrated wages and profits. According to Ricardo and to Marx, the centre of gravitation is given, in a first approximation, by the direct and indirect labour requirements.[4] We may write relative prices according to Shaikh (1976).

$$\frac{p_i}{p_j} = \frac{w_i' + \pi_i'}{w_j' + \pi_j'}$$

Since $l(I-A)^{-1} = \Lambda$ is the vector of direct and indirect labour requirements, we get the following relation:

$$\frac{p_i}{p_j} = \frac{\lambda_i(1+\pi_i'/w_i')}{\lambda_j(1+\pi_j'/w_j')}$$

The relative prices are determined by relative direct and indirect labour requirements and another term, which reflects the income distribution, not necessarily determined by the supply and demand mechanism. Ricardo, especially in his later writings, analysed how relative prices are perturbed by changes in income distribution between labour and capital. However, in his view the labour-embodied theory was still an adequate first approximation to the theory of value and a sufficient first-determination of the change of the centres of gravitation. It was seen that capital accumulation and technical progress produce changes in the natural price or the direct and indirect labour requirements. The direct and indirect labour requirements change, when the productivity of labour is increased or decreased. Thus the productivity of labour is, in a first approximation, the determinant of the centres of gravitation. The additional perturbation arising from a change in income distribution was thought to be small. In part II of the paper, these effects are discussed further.

(3) As also shown recently in many studies (Garegnani, 1983; Benetti, 1981; Cartelier, 1981; Bharadway, 1983) the classical theory of long-run price is not based on a supply and demand theory of price. Natural prices, natural wages, and natural profits should not be interpreted from the equilibrium concept of price found in general competitive analysis. Smith for example, does not speak about equilibrium prices or wage and profit rates but refers to "normal" or "average" prices, wage and profit. The limited role of supply and demand in the classics has three reasons. First, demand is thought to be given in the short-run (in Smith and Ricardo effectual demand, in Marx social demand, determined by the income distribution). Accidental and not persistent changes in demand affects only the market prices, not the natural price. Of course,

[4]Ricardo assumes a disturbance effect due to a change in the wage/profit rate relationship of 6 to 7 per cent. He argues that changes in relative prices are much more sensitive to changes in values than to changes in the distribution of income (Ricardo, 1951, ch. 1, sect. 5).

$(I-A)^{-1}$ is the Leontief inverse which, multiplied by wl, gives us the vertically integrated wages and, multiplied by rpB, gives us the vertically integrated profits. B is long-run changes of demand may lead to facility or difficulty to produce and thus can change the relative long-run centres of gavitation. Thus demand will in this way influence relative prices for example, due to increasing returns to scale in industrial production, or due to nonreproducible inputs. Also in the case of joint production long-run changes in demand will affect relative prices if a new technique is required to produce the products being demanded. Yet, in a first approximation, for the determination of the centres of gravity, demand is thought to be given (Garegnani, 1983; Benetti, 1981; Deleplace, 1981). Hence we could say that the classics assumed, as we might say by referring to modern mathematical techniques, a piecewise linear economic system. The normal conditions are taken as given and the variations around the normal conditions, for example, the variations of demand around the normal one, will not affect long-run relative prices. Secondly, although supply and demand are thought to be determinants of the market prices of the commodities, demand and supply analysis is not extended to labour and capital income (see for example the classical theory of wage determination). Thus, the components of long-run production prices are not determined by the supply and demand forces. In this sense many authors recently referred correctly to the classical concept as one containing two laws: one that determines the centres of gravitation the other (supply and demand) exerts its influence on the fluctuation of the market prices. Third, even for the fluctuation of the market prices, supply and demand were thought not to be sufficient determinants. In Smith, Ricardo and also in Marx, supply and demand and other accidental forces such as random events, speculation, temporary price setting due to monopolistic conditions and mobility barriers of capital between industries, also influence the market prices.[5] On the other hand, it does not seem to be true for the classics — and certainly not for Marx — that supply and demand is equalised by price changes as it is true for the neoclassical adjustment process, where excess demand responds to relative prices, and the flexibility of prices equate supply and demand. Since, as has been recently shown, a stable price adjustment process, responding to supply and demand, is equivalent to a stable excess demand function (Jordan, 1983), this means that markets are always cleared in the long-run, an idea quite unknown among the classics.

3. *On the Marxian Theory: Competition and Disequilibrium*

The above mentioned essentials of the classical theory of competition can be found in pre-Smithian, Smithian and Ricardian economic theory. Leaving aside the similarities and differences of Marx's theory of the surplus compared with the one of the classics, I want to summarise briefly Marx's view of the last two points mentioned in section I.2. By stressing mainly the differences *vis-a-vis* Smith and Ricardo, Marx formulated a more general and dynamic theory of competition than the classics (see also Deleplace, 1981 and Shaikh, 1978, 1980).[6] In comparison with the classics,

[5]For prices of production models with price setting behaviour of firms, see Schwartz, 1965, Brody, 1974, and Krause, 1983.

[6]For an excellent treatment of the Marxian theory of competition, see Kuruma (1977).

competition in Marx is a much broader concept. It is "competition of capitals" (see also Hollander, 1980), in the sense of rivalry among large capitalist firms, resulting from the goal of the firms to grow and to expand. Hence, competition of capitals is seen to affect the production process, circulation and investment flows. In production, the goal of capitalist competition is to create surplus profit through the use of new techniques and the increase of productivity of labour. Marx maintained that the "battle of competition" is fought by "cheapening the commodities (see also Shaikh, 1980 and Semmler, 1983). In circulation, the competition of capitals aims at enlarging the market share and improving the conditions for the realisation of profits. The intersectoral competition of capitals results in investment flows, bringing about a tendency to equalise the rates of profit across industries. As can be seen, competition in Marx is not equivalent to the equalisation of profit rates, or to price and quantity adjustments, but is related to capital accumulation and growth of the firm. Thus in Marx more so than in the classics, competition is seen not just as equilibrating force, but is viewed mainly as a force that produces disequilibria, distortions, and misallocations of resources. Marx speaks about the anarchy of the market, which adjusts through crises. In addition, competition of capitals leads to the downfall of firms, centralisation, and the rise of new firms. Thus, whereas in the classic especially (in Smith) competition is partly seen as an equilibrating force and in the neoclassical view, competition is regarded as a process of convergence toward an equilibrium, competition in Marx is a process of rivalry creating differentials in profitabilities and disequilibria. This concept is very close to the theory of competition in the Austrian tradition (Schumpeter) where competition is viewed as a process of creative destruction. Since for Marx there are not sufficient self-adjusting forces in a market system, he has a more stochastic concept of economic laws. He speaks of the "domination of the regulating averages" (Marx, 1977, p. 860). This does not refer necessarily to the average — or moving average — of the market prices or market rates of profit, but to the average conditions of production and demand. Thus, for production he maintains that due to competition of firms there is a co-existence of different techniques in industries and the social value or market value of the commodities is determined by the average production conditions of industries. The socially necessary technique, *i.e.* the regulating technique, is composed of the weighted average of the individual techniques (see Flaschel, 1983; Murata, 1977). On the other hand demand is seen to be given as "social demand", determined by the income distribution. However, since Marx does not assume constant returns to scale when analysing long-run production prices, long-run changes in demand may shift the centres of gravitation to the more or least efficient techniques in industries. Yet, similar to the classics, in demonstrating the fluctuation of the market price around the long-run production price, he also assumes a "piecewise linear economic system", where the fluctuations for demand around the normal one do not influence the centres of gravitation. Capital accumulation and technical change bring about the long-run change of the centres of gravitation either as direct and indirect labour requirements (market values) or as prices of production. In Marx, the prices of production are the more concrete regulating centres for the market prices, when commodities are the product of capital and when there are no mobility barriers for the movement of capital. The more concrete long-run centres of gravitation are given by the average

cost of production and the average rate of profit[7] on capital advanced. However, he maintained that the long-run change of the production price is more determined by the change of the direct and indirect labour requirement than by the perturbation of the production price arising from a change in the income distribution[8] (see part II).

Since the concept of centres of gravitation is not an equilibrium concept, fluctuations of supply and demand and oscillation of the market prices around the prices of production, and the actual or industry profit rates around the average profit rate, are considered the normal state of an economy. For the relation of supply and demand Marx maintains, that "supply and demand never equal(s) one another in a certain period, but only as an average of past movements, and only as the continuous movement of their contradiction" (Marx, 1977, p. 190). As seen, market phenomena are considered more as random events, kept within certain limits. Supply and demand are not equated by (market) price change as his analysis in vol. II of capital shows (see Foley, 1983) and (market) prices are not solely determined by supply and demand conditions (for a prices of production model with arbitrary mark-up pricing see Brody, 1974; Semmler, 1984 and Krause, 1983). Moreover since competition leads to the differentiation of production and market conditions, there is not only a tendency toward the equalisation of profit rates, but also a tendency toward the existence of differential profit rates among industries. Moreover the competition among firms within one industry and the co-existence of multiple techniques within one industry brings about differential of profit rates among firms even in the same industry (Flaschel, 1983; Murata, 1977). In addition, in the process of competition the price of the same commodity produced by different firms will not be the same (Marx, 1977, p. 193).[9] Marx did not assume (especially in Chapter X in Marx, 1977), that profit rates will be equalised among firms and industries. The process of competition among capitals produces differentials in profit rates as well as an equilibrating tendency.[10] As Marx put it: "The general rate of profit is never anything more than a tendency, a movement to equalise specific rates of profit." (Marx, 1977, p. 366) and "The average rate of profit does not obtain as directly established fact, but rather is to be determined as an end result of the equalisation of opposite fluctuations." (Marx, 1977, p. 368). However, allowing for the existence of differential profit rates among firms and industries one might face the objection that there cannot be a tendency toward prices of production as centres of gravitation.[11] Yet, as can be shown prices of production can

[7] Whether or not there is a convergence of market prices toward prices of production due to a formation of a general profit rate is discussed in Nikaido (1977), Flaschel (1983), Levy and Dumenil (1983).

[8] Marx speaks of the law of value which determines price movements indirectly, when prices of production prevail, see Marx, 1977, p. 175. For Marx's notion of centre of gravitation see Marx, 1977, p. 198.

[9] The different causes for differential profit rates are discussed further in Semmler (1984, ch. 4) where empirical evidence for differentials in profitability is also presented.

[10] Ricardo and Smith also speak of differentials of profit rates, when the market price deviates from the natural price for a considerably long time, see Ricardo (1951, ch. IV), Smith (1961, ch. VII).

[11] Against the assumption of the existence of differential profit rates, the objection has been made that prices are not determined any more by the reproduction cost and in case of structural barriers to entry differential profit rates cannot be used as guide posts for capital mobility. The empirical evidence, however, shows that we can have differential profit rates for a long time and commodities are produced and reproduced under these conditions. Moreover, barriers to entry — or barriers to mobility of capital — are not permanent barriers in the long-run. Barriers to entry are subject to many changes and can be overcome in the course of time. See Semmler (1984, ch. 5).

be formulated also under the condition of the existence of differentials in profitability among firms (Flaschel, 1983) and industries (Semmler, 1984). Here, this question cannot be pursued further, in the remaining part, some problems will be addressed, that arise with regard to the change of the centres of gravitation over time.

II. On the Change of the Centres of Gravitation

By modelling the classical and Marxian theory in linear production models, as is a common practice, we may face some other problems which are not discussed sufficiently in the literature. One of the main problems is related to the extent to which the centres of gravitation change, when the structure of production or income distribution change. In what follows mathematical sensitivity and error analysis will be used to study the robustness of the centre of gravitation against perturbation of the structure of production or the income distribution. (A similar problem has recently been discussed in neoclassical economics (see Dierker, 1982). Here, we want to adopt the notion "structural change" and not "choice of technique", since the former allows also for a change of the structure of production which is not necessarily a cost minimising or profit rate maximising one. For the discussion on the choice of technique (which we want to leave out for reasons of simplicity), see Okishio (1961), Shaikh (1978), Roemer (1979), Lipietz (1980) and Semmler (1983). Thus the change of techniques as it is discussed in what follows, might not be a "viable" one, but this will not change the results as can be seen from the appendices. The following presentation, allows us also to discuss again the problem of the market mechanism, when structural changes occur. As already discussed in part I of the paper, the concept of centre of gravitation seems to imply that relative values λ_i/λ_j or relative prices of production p_i/p_j remain quite stable over time. Thus they seem to be very robust in that they are neither affected by short-run changes in supply and demand nor by small perturbation in the production conditions. Modern interpretations of the centre of gravitation concept have not, however, studied sufficiently the question how sensitive relative values or relative prices of production are to a perturbation in the production conditions when values or prices are modelled in linear production systems, and to what extent market mechanisms exist to make new centres of gravitation relevant ones. Of course, in the discussion, that refers to Sraffa-prices attempts have been made to demonstrate how relative prices change, if wages or the profit rate change (see Schefold, 1976). Yet, exact proofs of the magnitude and direction of the change in relative prices due to a change in the real wage or the profit rate have not been given. In addition, the problem of how relative values or prices of production alter due to a change in the coefficients of production (usually represented in the matrix A and the labour vector l) has not been addressed. In the following, for reasons of simplicity, we will work with a circulating capital model, to demonstrate the effect of some perturbations. A model, including fixed capital, can be found in Semmler (1984).

1. *Change of Values Due to a Change in the Structure of Production*

We may use the following example to discuss the effect of changes in the coefficients of production on relative values and prices of production. A linear system,

representing only a circulating capital model, from which values can be derived is usually written in the following form:

$$\Lambda = \Lambda A + l$$

where Λ is the value vector, A the input-output matrix in physical terms and l the vector of direct labour coefficients[12] (for the following example, see Pasinetti, 1977, p. 144).

We assume that

$$A = \begin{bmatrix} \frac{186}{450} & \frac{54}{21} & \frac{30}{60} \\ \frac{12}{450} & \frac{6}{21} & \frac{3}{60} \\ \frac{9}{450} & \frac{6}{21} & \frac{15}{60} \end{bmatrix}$$

and that the labour coefficients are

$$l = \begin{bmatrix} \frac{18}{450} & \frac{12}{21} & \frac{30}{60} \end{bmatrix}$$

We derive the following values

$$\Lambda = l(I-A)^{-1} = (0.1818 \quad 1.81818 \quad 0.90909)$$

We assume a real wage vector

$$d = \begin{bmatrix} 2 \\ 0 \\ 0.1666 \end{bmatrix}$$

The value of labour power is

$$v = \Lambda d = (0.1818 \quad 1.81818 \quad 0.90909) \begin{bmatrix} 2 \\ 0 \\ 0.1666 \end{bmatrix} = 0.515$$

And the rate of surplus value is

$$e = \frac{1-v}{v} = 0.9411$$

Now we assume, that technical change or — in more general terms — structural change, occurs due to capital accumulation. Let us first consider the case where one element of the A matrix changes, for example the second production process may use up more inputs than before. This could be taken out of the net product of the economic system

[12]Since Sraffa-prices can be written as $p = pA(1+r) + wl$ or $p_w = l(I-(1+r)A)^{-1}$ the following discussion of the influence of change in production coefficients on relative values is also applicable to Sraffa's price concept, where p_w are prices in terms of labour commanded. Schefold (1976), who shows that prices change with a "different speed", refers to the sensitive price change due only to a change in the rate of profit r.

or provided by additional production of the first production process. In this case a_{12} would increase without any change in the other coefficients of the matrix A or the vector l, *i.e.* since the vector for the coefficients of direct labour does not change, the organic composition of capital for the second production process would rise only due to an increase of constant capital that is used up in the second production process. Hence, a change in the value vector would stem solely from a change in the material matrix A. An increase in a_{12} of A would lead to an increase in the elements of the value vector Λ. But we want to discuss a more general case, where coefficients for constant capital and labour coefficients change. We assume that industry 1 produces more output to match the increase of inputs in industry 2 and that the coefficients for the first production process a_{i1} $(i = 1,2,3)$ and l may alter too. We get the following system now:

$$(\tilde{\lambda}_1 \tilde{\lambda}_2 \tilde{\lambda}_3) = (\tilde{l}_1 l_2 l_3) \left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} \tilde{a}_{11} \tilde{a}_{12} a_{13} \\ \tilde{a}_{21} a_{22} a_{23} \\ \tilde{a}_{31} a_{23} a_{33} \end{bmatrix} \right\}^{-1}$$

We intuitively can realise that the relative values may be quite sensitive to changes in the organic composition of capital. The magnitude of the change depends on the change in l and the change in the inverse $(I - A)^{-1}$ and a slight change in A may have a strong effect on the new solution $\tilde{\Lambda}$.[13]

This for example, is quite obvious if the inverse is almost singular before the alteration of the elements of A and becomes singular after a slight alteration of the elements in A. Moreover, already a change of one element in A affects all elements in the inverse $(I - A)^{-1}$ and sensitive changes can occur for certain types of matrices. These changes also affect the rate of exploitation because Λd is affected, even though d is not altered. Since $\Lambda(I - A) = l$ can be written as $\Lambda B = l$, we can demonstrate in general the change of the solution of the system if we assume a disturbance of the original matrix A as well as a disturbance of the vector l.

The theory of error analysis, used in numerical methods can be applied to this problem. We can write the disturbance effect in the following form:

$$(\Lambda + \delta\Lambda) \quad (B + \delta B) = l + \delta l$$

which can be written as

$$\tilde{\Lambda} = (l + \delta l) \quad (B + \delta B)^{-1}$$

where Λ is the original value vector, B the matrix $(I - A)$, $\delta\Lambda$ the change in the solution of the equation system, $\tilde{\Lambda}$ the new solution, δB the perturbation in the matrix B and δl the perturbation in the vector l. The sign for the perturbed matrix B and l can be positive or negative. Thus δ is the variation by which the coefficients can change due

[13] The possible strong effect on relative values due to change in coefficients does not necessarily mean that values change discontinuously due to a small change in coefficients, but rather that values change at "different speed".

to technical or structural change. The most sensitive part of the solution of $\tilde{\Lambda}$ is the chance in the inverse $(B + \delta B)^{-1}$. As mentioned, the inverse can even become singular due to a slight change in the elements of the matrix A. In this case we will have no solution. From the formula above we can derive the perturbation effect of the value vector:

$$\delta\Lambda = (\delta l - \Lambda \delta B) B^{-1} \quad (I + \delta B B^{-1})^{-1}$$

A general estimation of the disturbances of the inverse and thus the disturbance of the vector Λ due to δB and δl is given in the Appendix 1.

How sensitive a solution of a linear system may be to slight perturbation of B for certain matrices is discussed in Appendix 1. There are, of course, some limits for the range of the change of the average production coefficients in industries. In Marx for example, there is the role of firms in industries — firms with more efficient techniques, average technique and least efficient technique — that would at least dampen the impact of changes in coefficients of firms on the average conditions in the industries and thus on the change of relative values (see Marx, 1977, ch. IX and X). On the other hand values are conceptualised as market values which reflect average conditions of productions and demand over a certain length of time where accidental perturbations do not seem to have such a disturbing influence on relative values as centres of gravitation for prices. Yet, as demonstrated in the appendices, the change of the solution does not depend so much on the magnitude in the change of the coefficients but more on the type of matrices. Therefore, with regard to the value vector derived from a linear production model, we can see that the centres of gravitation around which the market price is supposed to fluctuate, can themselves change quite strongly. This would raise, of course, the problem of an economic adjustment process, *i.e.*, the working of the market process, that allows the new values to become the new relevant centres of gravitation for actual prices. Before we draw some further conclusions, we want to show that a similar strong perturbation can occur in a prices of production system.

2. *Change of Prices of Production due to Changes in the Structure of Production and Income Distribution*

If we use linear production models to derive the average profit rate and prices of production across the industries we can get similar results. In the sense of Marx, prices of production are the more concrete centres of gravitation for market prices brought about by competition.[14] The system of prices of production is usually written in the following form: $p(A + dl) = \lambda p$ or in the form of the transpose of $(A + dl)$: $\bar{A}'p = \lambda p$, where λ is the maximum eigenvalue of the indecomposible matrix $\bar{A}'$ and p is the eigenvector associated with this maximum eigenvalue. If we use our example,

[14] The following considerations can also be applied to the Sraffa's framework. The standard commodity in Sraffa is calculated from the following eigenvalue system $(1 + R)A\,X = X$ (single product system). The discussion in II.2 develops a method to estimate the change in the standard output vector X due to a change in coefficients.

introduced above, the system of prices of production can be written in the following form:

$$\left(\begin{bmatrix} \frac{186}{450} & \frac{54}{21} & \frac{30}{60} \\ \frac{12}{450} & \frac{6}{21} & \frac{3}{60} \\ \frac{9}{450} & \frac{6}{21} & \frac{15}{60} \end{bmatrix} + \begin{bmatrix} \frac{36}{450} & \frac{24}{12} & \frac{60}{60} \\ 0 & 0 & 0 \\ \frac{3}{450} & \frac{2}{21} & \frac{5}{60} \end{bmatrix}\right) \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} = \lambda \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}$$

The maximum eigenvalue λ is 0.8436.

If the value of the net product $pdl\ (l+c)=1$ is taken as *numeraire*, we get the following vector for prices of production associated with $\lambda(\bar{A}')$.

$$p=(0.1950 \quad 0.1809 \quad 0.750)$$

Starting from this example, we now discuss three cases. A more detailed analysis is given in Appendix 2.

(1) If due to technical change elements of a_{ij} in the matrix A alter and therefore the proportion of capital to labour changes, not only will the value vector change but also the profit rate and the prices of production. As is well known from the Frobenius theorem, the profit rate is a continuous function of the elements of A; however, as is shown in Appendix 2, the direction and magnitude of the change of relative prices of production cannot easily be so determined.[15] What can be shown is that for certain technical conditions of production, *i.e.*, for certain matrices, the relative prices can change quite sensitively due to small changes in the elements of matrix A.

(2) Furthermore, we can see that the prices of production and the profit rate will alter when the real wage vector d changes. For example, if the real wage bundle increases, the profit rate will decline and the elements of the vector of prices of production will change too (see Appendix 2).[16] But in this case, the value vector remains the same, since neither l nor A is altered. We only get a disturbance of the prices of production (and the profit rate). The values remain the same. This is a case referred to by Ricardo and Marx when they discussed the influence of wages on the values of commodities. They showed, contrary to Smith, that the values of commodities are not influenced by a change in wages.

(3) If the proportion of capital to labour is altered due to a change in the elements of A and l is also altered — *i.e.*, A and l in the matrix $(A+dl)$ is altered — the values as well as the prices of production will change, but the magnitude and the direction of both changes are quite unclear (see Appendix 2).

[15] It is true that the eigenvalue and thus the profit rate is a continuous function of $\bar{A}$, but in case of an ill-conditioned eigenvalue it can change sensitively in response to a small perturbation of $\bar{A}$. See Ortega (1962, p. 44).

For changes of prices of production and the profit rate due to a disturbance of the matrix A and $\bar{A}$, we can develop a general estimation. Using a similar approach as before, we can separate the disturbance effect on λ and p due to a disturbance in $\bar{A}'$ and write:

$$(\bar{A}'+\delta\bar{A}')(p^{(r)}+\delta p^{(r)})=(\lambda_r+\delta\lambda_r)(p^{(r)}+\delta p^{(r)})$$

where $\delta\bar{A}'$ is the disturbance of the matrix $\bar{A}'$, $p^{(r)}$ and $\delta p^{(r)}$ the original and disturbed eigenvector, λ_r and $\delta\lambda_r$ the original and disturbed eigenvalue. An estimation of this disturbance effect is developed in the Appendix 2.

As shown in Fox (1965, p. 275) and in Appendix 2, in the case of a symmetric matrix $\bar{A}'$, a small perturbation of the elements of the matrix $\bar{A}'$, does not affect the eigenvalues greatly, whereas in the case of an unsymmetric matrix a great change of the eigenvalues (and the maximum eigenvalues) is possible due to a change in the matrix $\bar{A}'$. In either case, if all eigenvalues of the matrix $\bar{A}'$ are very close to each other, the eigenvector of the disturbed matrix $\tilde{A}'$ can be disturbed greatly, $\tilde{A}'=(\bar{A}+\delta\bar{A})'$. In other words, even if the maximum profit rate $r(\bar{A}')$ is not disturbed very much by a perturbation of a matrix $\bar{A}'$ to matrix $\tilde{A}'$, the prices of production associated with the new maximum profit rate $r.(\tilde{A}')$ may undergo considerable changes. Thus we can see for the system of prices of production for certain types of matrices a similar possible disturbance effect as we already demonstrated for the system of values. However, the classics and Marx seem to maintain that the centres of gravitation do not show these effects. In the long-run, due to accumulation and technical change, there is a change in the average conditions of production (see Marx, 1977, ch. IX and X) and it was maintained, especially by Marx, that changes in the long-run production conditions dominate the impact of the change of income distribution on relative prices of production.[17] However, for a given period of time, it was assumed that direct and indirect labour requirements, prices of production and the general profit rate are thought to be quite stable and are not sensitive to small changes in demand or structure of production (see Ricardo's debate with Malthus, presented in Martin 1982, and

[16] However, it is possible to show that there exists an upper limit for the price change due to a change in the real wage vector d (see appendix 2). By what speed prices of production — measured in any *numéraire* — change, when the income distribution changes, remains a question still to be solved. To what extent prices in a large economic system switch over when the income distribution (the vector d) is altered, still has to be further discussed. It is possible that the problem of switching of prices in a large economic system, where different price effects due to change of different variables may cancel out to a certain extent might not be so severe. A first attempt has been made by Schefold (1976) who discusses the change of relative prices due to a change in the profit rate for Sraffa-prices, where wages are paid *ex post*.

[17] Marx speaks of an indirect regulation of relative prices by values (see footnote 8). This proof of the empirical relevance of the labour theory of value that maintains that the change in direct and indirect labour requirements are more important for relative price changes than the change in income distribution, was first attacked by Böhm-Bawerk. An exact proof has not been given for this Marxian theorem, but neither has there been an exact counterproof, since it is very difficult to estimate the change of relative prices (prices of production) due to a simultaneous change of the labour coefficients and wages (or income distribution). However, some empirical evidence on the dominance of the change in productivity over the change of other input costs for relative price change can be found in Houthakker (1979) and Semmler (1984).

Marx 1977, p. 190 and p. 860).[18] Some further conclusions that can be drawn from this analysis are presented below.

III. Some Conclusions

As shown, many authors recently seem to agree that the classical analysis of the market did not rest on assumptions of a perfectly competitive economy. This paper also provides much doubt, as to whether in the classics, the competitive process has been regarded to be as strong an equilibrating process as in neoclassical general competitive analysis. In classical economics, demand and supply analysis played a very limited role; market mechanisms which provide a convergence of actual prices toward equilibrium prices while simultaneously equilibrating supply and demand are more characteristic of neoclassical than classical economics. Especially, as shown, within the context of the Marxian dynamic theory of competition, it would be very peculiar to assume stable market processes with such dual properties. A second set of problems with regard to, not the perturbation of prices and quantities, but to production coefficients and income distribution have been discussed. We have shown that under certain conditions, the centres of gravitation (direct and indirect labour requirements or prices of production) can change themselves quite strongly, when modelled in linear production systems. In recent literature on dynamical systems, it has been demonstrated that an irregular behaviour of solutions will occur more frequently in non-linear systems than in linear ones (Hirsh and Smale 1974, ch. 16 and Varian 1981). Yet, linear production models, which are usually used to depict the classical and Marxian theory seem already to exhibit such properties when certain types of matrices are allowed for. As mentioned this leaves us with somewhat unsatisfactory conclusions. We might conclude that we have to look for a more complex treatment of the classics and Marx, where long-run production prices can be shown to be quite robust concerning structural changes. This however, would mean that the properties of the classics and Marx are not depicted well in a linear production model. On the other hand we might also conclude that they are well represented in linear production models but that the distance from one set of production prices to another is, under certain conditions, too great to be bridged by any known market mechanism. The time requirements to reach the new production prices might be too great thus adjustment mechanisms might not be conceivable, which could lead to the new centres of gravitation.[19] We also might conclude that the new centres of gravitation

[18] Concerning the law regulating long-run prices, Marx maintains: "Under capitalist production, the general law acts as the prevailing tendency only in a very complicated and appropriate manner, as a never ascertainable average of ceaseless fluctuations" (Marx, 1977, p. 161). Another mathematical concept might have to be worked out in order to be able to model the Marxian theory of competition, value and price correctly. See Lange (1963, ch. 3), where he speaks of the "stochastic character" of economic laws. Marx speaks about "opposing movements" with regard to coefficients in industries as well as with regard to prices, profits and wages. He uses this concept to demonstrate their relative stability in a large economic system where the different effects of the changes in different variables tend to cancel each other out in the short run, see Marx (1977, ch. X).

[19] The estimation discussed above also can be used to analyse the possible deviations of values from prices of production, if both the value system and the prices of production system are formulated in terms of an eigenvalue problem.

are indeed relevant ones, but they can be reached only by "catastrophic" events or severe disruptions. Therefore, there are, as shown, different possible conclusions and future research may show which conclusions are the more reasonable ones.

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Appendix I

For a linear equation system $\Lambda B = l$, presented in part II, the solution Λ is determined by the inverse B^{-1} since $\Lambda = lB^{-1}$. The question is how the inverse changes if B changes to $B + \delta B$. We write $\delta B = E$, where E is a disturbance of B. We might examine the maximum error of the new inverse and the effect on the solution of $\bar{\Lambda}$ by the following method (see Fox, 1965, p. 141).

We can take $(B + E) - B = E$ and pre- and postmultiply these equations by $(B + E)^{-1}$ and B^{-1}. We get $B^{-1} - (B + E)^{-1} = (B + E)^{-1} EB^{-1}$. In order to get an estimation for the maximum change in the new inverse we take norms and use the fact that $(B + E)^{-1} = (I + B^{-1}E)^{-1}B^{-1}$.

We get

$$\| B^{-1} - (B + E)^{-1} \| \leqslant \frac{\| B^{-1} \|^2 \| E \|}{1 - \| B^{-1} \| \cdot \| E \|}$$

(assuming that $\| B^{-1} \| \cdot \| E \| < 1$)

Multiplying the numerator and denominator on the right hand side by $\| B \| / \| B \|$ and dividing the whole equation by $\| B^{-1} \|$ we get the following expression:

$$\frac{\| B^{-1} - (B + E)^{-1} \|}{\| B^{-1} \|} \quad \frac{\| B^{-1} \| \cdot \| B \| \cdot \| E \| / \| B \|}{1 - \| B^{-1} \| \cdot \| B \| \cdot \| E \| / \| B \|}$$

We can see the relative disturbance of the original matrix B (due to the change in the organic composition of our original matrix A) is determined by $\| B^{-1} \| \cdot \| B \|$ and $\| E \| / \| B \|$. If $\| B^{-1} \| \cdot \| B \|$ is small, the relative disturbance of the solution will also be small. If $\| B^{-1} \| \cdot \| B \|$ is large, it may no longer be concluded that the disturbance will be small. In the literature types of matrices are discussed where the disturbances are very large (see Fox, 1965; Ortega, 1962; Voievodine, 1980). On the other hand, a strong disturbance E will also change the solution greatly. But the effect on the new solution is already determined by the original matrix B. Of course, as mentioned in the text, the solution of $\bar{\Lambda}$ is also influenced if the vector l also changes.

Since $\bar{\Lambda}(B + E) = l + \delta l$, a possible disturbance of l (the direct labour coefficients) has also an influence on the solution $\bar{\Lambda}$, since $\bar{\Lambda} = (l + \delta l)(B + E)^{-1}$, or in other words, the degree of ill-conditioning of the matrix B, i.e., of the matrix A. By using the formula $\delta\Lambda = (\delta l - \Lambda\,\delta B)B^{-1}(I + \delta BB^{-1})^{-1}$ from part II, we also can estimate the relative change in the vector for the values. By taking norms of this equation and using the upper bound for the inverse

$$\| (I + \delta BB^{-1})^{-1} \| \leqslant \frac{1}{1 - \| \delta B \| \cdot \| B^{-1} \|}, \text{ if } \| \delta B \| \cdot \| B^{-1} \| < 1$$

and dividing both sides of the equation by $\| \Lambda \|$ we get:

$$\frac{\| \delta\Lambda \|}{\| \Lambda \|} \leqslant \left(\frac{\| \delta l \|}{\| \Lambda \|} - \| \delta B \| \right) \cdot \frac{\| B^{-1} \|}{1 - \| \delta B \| \cdot \| B^{-1} \|}$$

Λ is usually a continuous function of B, but this does not say anything about the size of the change $\delta\Lambda$. This is determined by the degree of ill-conditioning of the matrix B. This effect occurs, for example, if there are almost linear dependent rows or columns, see Fox (1965, p. 139). This phenomenon is also well known in non-linear systems, see Varian (1977)

An example for a linear system may be the following one:

$$\left\{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.3 & 0.699 \\ 0.7 & 0.3 \end{bmatrix}\right\} = (0.1 \quad 0.1)$$

The matrix for the material inputs as well as the labour vector show positive elements and the matrix is productive. The solution of the system

$$\Lambda \begin{bmatrix} 0.7 & -0.699 \\ -0.7 & 0.7 \end{bmatrix} = (0.1 \quad 0.1)$$

is $\lambda_1 = 200$ and $\lambda_2 = 199.86$

Now we change one element in the matrix for material inputs. We want to solve the following system:

$$\begin{bmatrix} 0.7 & -0.6999 \\ -0.7 & 0.7 \end{bmatrix} = (0.1 \quad 0.1)$$

We get the following solution $\lambda_1 = 2000$ and $\lambda_2 = 1999.29$

Another example, in which the original matrix A is also productive, may be:

$$\begin{bmatrix} 0.7700 & -0.5000 & -0.3333 \\ -0.5000 & 0.6667 & -0.2500 \\ -0.3333 & -0.2500 & 0.8000 \end{bmatrix} = (0.8 \quad 0.4 \quad 0.6)$$

The solution of the system is:

$$\lambda_1 = 150.427, \quad \lambda_2 = 155.407, \quad \lambda_3 = 111.986$$

However, if we change the b_{21} from -0.2500 to -0.2400 we get the following solution:

$$\lambda_1 = 88.679, \quad \lambda_2 = 90.903, \quad \lambda_3 = 66.103$$

In this case as well as in the first case it may be argued that the relative values only change slightly whereas the absolute values change very considerably. However, from an economic point of view, the second property (considerable change of prices, even if the change is a continuous function of the coefficients) already creates serious problems. If we assume for example — as Ricardo did (Ricardo, 1951, ch. VII) that gold is produced outside the country and that one unit of labour is embodied in one ounce of gold, then the prices of the country, in which the production coefficients have changed slightly would change considerably. This would also change the absolute or comparative cost advantage of a country and lead to a new international division of labour (depending upon what mechanism is assumed to establish the new absolute or relative cost advantage). However, it is hard to conceive an economic mechanism that could lead to an adjustment toward the new price level and toward a new international pattern of trade (see also Steedman, 1979). The same problem of an economic adjustment will arise if relative values change.

APPENDIX II

The equation $(\bar{A}' + \delta\bar{A}')(p^{(r)} + \delta p^{(r)}) = (\lambda_r + \delta\lambda_r)(p^{(r)} + \delta p^{(r)})$ in part II can be written as $\tilde{A}'\tilde{p}^{(r)} = \tilde{\lambda}_r\tilde{p}^{(r)}$ (1), where $\tilde{A}'$ is the changed matrix $\bar{A}'$, $\tilde{\lambda}_r$ the new maximum eigenvalue and $\tilde{p}^{(r)}$ the corresponding eigenvector which can be interpreted as the new vector of prices of production. In order to simplify the following derivations, we take for $\bar{A}' + \delta\bar{A}'$ the expression $A + \delta A$ and write for δA the term ϵB, where ϵ is a small number and B a matrix (see Fox, 1965, pp. 277). Thus, we can write (1) as

$$(A + \epsilon B)p^{(r)}(\epsilon) = \lambda_r(\epsilon)p^{(r)}(\epsilon) \; . \qquad (1')$$

On the other side for the original structure of production — represented by the matrix $\bar{A}'$ — we get the

following prices of production and the following eigenvalue (dropping the indices again)

$$Ap^{(r)} = \lambda_r p^{(r)} \qquad (2)$$

We know that all eigenvalues of the matrix A can be expressed by the similarity transformation of the matrix A, *i.e.* we know that if A can be transformed into a diagonal matrix, we get

$$X'AP = \Lambda \qquad (2')$$

where $X' = P^{-1}$. X' is the transposed matrix of the lefthand eigenvectors of A and P the matrix of the right-hand eigenvectors of A corresponding to the eigenvalues Λ. (Λ is a diagonal matrix of the eigenvalues $\lambda_1, \lambda_2, \ldots\ldots, \lambda_n$). The individual vectors of X' and P can be normalised so that $x^{(r)'}x^{(r)} = 1$ and $p^{(r)'}p^{(r)} = 1$. For a symmetric matrix A we get $x^{(r)'}p^{(r)} = p^{(r)'}p^{(r)} = 1$. For a more general matrix (nonsymmetric matrix A), we can write $x^{(r)'}p^{(r)} = q_r$.

For the perturbed matrix $\tilde{A} = A + \epsilon B$ we get the following general similarity transformation:

$$X'(A + \epsilon B)P = \Lambda + \epsilon C \qquad (3)$$

where the element of c_{ij} of C is given by $x^{(i)'}Bp^{(j)}/q_i$. This is true since $X'AP = \Lambda$.

From (1') we know that $p^r(\epsilon)$ is an eigenvector of $(A + \epsilon B)$ associated with the eigenvalue $\lambda_r(\epsilon)$. An eigenvector of the system (3) can be written as $z^{(r)}(\epsilon)$, so that we get

$$X'(A + \epsilon B)Pz^{(r)}(\epsilon) = (\Lambda + \epsilon C)z^{(r)}(\epsilon) = \lambda_r(\epsilon)z^{(r)}(\epsilon)$$

and

$$p^{(r)}(\epsilon) = Pz^{(r)}(\epsilon) \; . \qquad (4)$$

(4) is equal to (1'), since we can substitute for $z^{(r)}(\epsilon)$ in (4) the expressions $P^{-1}p^{(r)}(\epsilon)$. Thus we can write for (4)

$$X'(A + \epsilon B)PP^{-1}p^{(r)}(\epsilon) = \lambda_r(\epsilon)P^{-1}p^{(r)}(\epsilon) \; .$$

Since we know that $X' = P^{-1}$ we get (1')

$$(A + \epsilon B)p^{(r)}(\epsilon) = \lambda_r(\epsilon)p^{(r)}(\epsilon) \; .$$

Moreover we assume the rth component of $z^{(r)}(\epsilon)$ is largest and we normalise so that this component is unity. Then according to equation (4) for the sth component of $z^{(r)}(\epsilon)$, with $s \neq r$ we get the result

$$\lambda_r(\epsilon)z_s^{(r)}(\epsilon) = \lambda_s z_s^{(r)}(\epsilon) + \epsilon \sum_{i=1}^{n} c_{si} z_i^{(r)}(\epsilon) \qquad (4')$$

Since the components of $z^{(r)}$ are equal or less than unity and $c_{si} = x^{(s)'}Bp^{(i)}/q_s$ (see (3)) we get the following result

$$|\lambda_r(\epsilon) - \lambda_s| \; |z_s^{(r)}(\epsilon)| \leqslant \epsilon |q_s^{-1}| \sum_{i=1}^{n} |x^{(s)'}Bp^{(i)}| \qquad (4'')$$

From (4'') we get

$$z_s^{(r)}(\epsilon) \leqslant \frac{\epsilon |q_s^{-1}| \sum_{i=1}^{n} |x^{(s)}Bp^{(i)}|}{|\lambda_r(\epsilon) - \lambda_s|} \qquad (4''')$$

We immediately can see that those components $z_s^r(\epsilon)$ may not be small any more for which a latent root (belonging to the undisturbed system of production $\bar{A}'$) is near to the $\lambda_r(\epsilon)$ corresponding to $z^{(r)}$. Moreover all the components $p^{(r)}(\epsilon) = Pz^{(r)}(\epsilon)$ may be disturbed badly. We see even if, for example, the new maximum eigenvalue $\lambda_r(\epsilon)$ for our disturbed matrix $\tilde{A}'$ is near the old maximum eigenvalue of the matrix $\bar{A}'$ (for example near λ_s) the new price vector $p^{(r)}(\epsilon)$ can be greatly affected.

From the Frobenius theorem we know that the maximum eigenvalue of an indecomposible matrix $\bar{A}'$ is increasing (decreasing) if the elements of the matrix $\bar{A}'$ increase (decrease), and the maximum profit rate falls (increases). But we do not know how much the eigenvector is disturbed. Formula (4''') gives us an estimation of such disturbance. (In a case where elements of the matrix $\bar{A}'$ change in different directions the change of the maximum eigenvalue can also be estimated, see Fox (1965, p. 276).

We can discuss now the three cases mentioned in part II of the paper.

(1) In case one where we assumed an increasing organic composition of capital, due to an increase of elements of the matrix A, ϵB can be regarded as $(\delta A)'$. According to (4''') we may get great disturbances of the price vector $p^{(t)}$ if the differences of latent roots $|\lambda_i(\epsilon) - \lambda_i|$ is very small and $|q_i^{-1}|$ differs from 1 greatly.

(2) In the second case, where the matrix $\bar{A}'$ changes due to a change in the real wage vector d, ϵB can be regarded as $(\delta dl)'$. The price change can be great, depending upon whether or not $\lambda_i(\epsilon)$ is near λ_i and $|q_i^{-1}|$ differs greatly from 1. But the fact that prices may change greatly does not depend so much on the change of d (represented by B in 4''') but more on the original matrix $\bar{A}'$ (the ill-conditioning of the vector problem)

(3) In the third case, if ϵB is $(\delta A + d\delta l)'$ the tolerance for the price change can be estimated from (4'''), but prices may change in different directions and different magnitude, depending again on the difference of $|\lambda_i(\epsilon) - \lambda_i|$, $|q_i^{-1}|$ and B. But here again $|q_i^{-1}|$ is already given by the matrix $\bar{A}'$, *i.e.* by the ill-conditioning of the matrix $\bar{A}'$. The change in the matrix $\epsilon B = (\delta A + d\delta l)'$ is of special interest for the discussion on choice of technique and falling rate of profit.[19] Usually by referring to the choice of technique criteria it is assumed that the new matrix $\bar{A}' = \bar{A}' + \epsilon B$ summed up with the old price vector will lead to a decreasing eigenvalue. Thus a higher profit rate (associated with a new price vector) will be the result (see Okishio, 1961; Roemer, 1979). However, this result is not true in general if we allow for a technical change where some elements of a matrix increase and some decrease. This can occur when a fixed capital matrix is included in the equation system (1). Due to the use of a new cost minimising technique, which affect the circulating capital, including depreciation, the profit margin on cost can rise, yet the profit rate on total fixed capital can decline (see Semmler, 1983).

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[10]

NATURAL PRICES, DIFFERENTIAL PROFIT RATES AND THE CLASSICAL COMPETITIVE PROCESS*

by

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Few would question the centrality in classical economic theory of the idea that the mobility of capital (and labour) between industries tends both to bring about a uniformity of rates of profit and to drive the market prices of commodities towards the corresponding natural prices. In the first section of this paper the presentations of this idea given by Smith, Ricardo and Marx are briefly reviewed, particular attention being paid to the way in which they associated a positive (negative) deviation of a commodity's market price from its natural price, with a positive (negative) deviation of the corresponding industry's profit rate from the natural rate. That there should be such a positive correlation of price deviations and profit rate deviations is not immediately obvious, since an industry's means of production will themselves be purchased at market, rather than natural, prices. Could it not happen, then, that an industry whose product's market price lies *above* its natural price, purchases as produced inputs commodities whose market prices lie "even more above" their natural prices, with the result that that industry has a profit rate *below* the natural rate? In the second section of this paper, it is demonstrated that it could indeed happen. In order to simplify the discussion and to focus on one issue at a time, however, that demonstration is given not in the context of a competitive process but in the context of an unchanging economy in which each industry earns a different rate of profit. Having shown that price deviations and profit rate deviations need not be in the same direction, one is naturally led to consider whether the competitive

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