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OF

PART V.—LIFE TABLES.

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Introduction.

One of the most valuable uses to which trustworthy rates of mortality can be put is for the construction of life tables; and it may further be asserted that the construction of life tables is almost necessary in order to give scientific value to death rates. The first extract in the following selections bearing upon the inter-relations of death-rates and life tables shows that even in the First Annual Report Dr. Farr fully appreciated the enormous value of the civil registration of deaths (showing the ages at death and the causes of death), not only as the basis of mortality statistics for stimulating sanitary progress, but as a means for the construction of true life tables, that would give increased trustworthiness to death-rates, to life insurance, and to the system of life annuities. It will be evident, to those who read these selections from his writings, how much thought and labour were devoted, from time to time, by Dr. Farr to the utilisation of the accumulating mortality statistics derived from the national Death Register for life table purposes. The first of Dr. Farr's English life tables, known as No. 1, was published in the Registrar General's 5th Annual Report, and was based upon the enumerated population in 1841 and the registered deaths in the same year. The following extracts include several from this report, in which appeared a general dissertation upon the history, construction, and properties of life tables, in addition to a detailed description of the English Life Table No. 1. This report also contained shortened life tables for the Metropolis, for Liverpool, and for the county of Surrey, which exemplified scientifically the effect of extreme urban and of rural death-rates upon the lifetime of a population.

In the Registrar General's 12th Report was published Dr. Farr's English Life Table No. 2, which was constructed upon a more extended basis, namely, the registered deaths in the seven years 1838–1844, and the Census enumerations in 1831 and 1841. The last of Dr. Farr's national life tables was based upon the recorded deaths in the 17 years 1838–54, and upon the three Census enumerations of population in 1831, 1841, and 1851. Such an extended series of observed facts had never before served as the basis of a life table. The near agreement of the results arrived at in these three English life tables is very remarkable, and accentuates the fact that English mortality, notwithstanding fluctuations from year to year, was, on the whole, notably stationary during the first 33 years of civil registration, 1838–70.

The English Life Table No. 3 was not only based upon a far larger series of observations than either of the Tables No. 1 or No. 2, but contained a far more complete and elaborate series of tables. This life table* was published as a separate volume, containing more than 500 pages, a great portion of which was devoted to joint life tables.

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* English Life Table. Tables of Lifetimes, Annuities, and Premiums; with an introduction by William Farr, M.D., F.R.S. Longman & Co.; 1864.

The limits of this volume forbade the selection of but comparatively few extracts of more general interest, and necessitated the omission of much possessing intrinsic mathematical and actuarial value. The main object of the selection was to take those portions of the reports which dealt with general principles and possessed value or interest for those who use the life table as a scientific exponent of mortality, rather than as a guide to insurance or annuity operations. It is necessary, therefore, to refer those who would study the more purely mathematical aspect of the question to the original reports in which the several life tables were published.

The Healthy District Life Table was based upon the mortality during the five years 1849–53, in 63 English registration districts which showed a mean annual death-rate not exceeding 17 per 1,000 persons living during the ten years 1841–50. This life table formed the subject of a Paper read by Dr. Farr before the Royal Society on 7 April, 1859, and printed in the Journal of the Transactions of the Society for that year. The tables based upon this Healthy District experience were afterwards added to and extended; and a selection from them was published in an Appendix to the Registrar General's 33rd Annual Report. Dr. Farr pointed out that this Healthy District Life Table expresses "very accu- "rately the actual duration of life among the clergy and other classes "of the community living under favourable circumstances." As representing, therefore, a standard of attained healthiness* (it is impossible, however, to say how much it is below an attainable standard), this healthy district mortality is most valuable as a means for measuring the excess of mortality due to insanitary condition, and generally to preventable causes.

The careful study of the following selection of extracts from Dr. Farr's writings on the construction and use of life tables cannot fail to convince all students of vital statistics that a life table supplies the only scientific and thoroughly trustworthy method for ascertaining the true import of rates of mortality in increasing or decreasing populations, whether such increase or decrease be due to the difference between birth-rates and death-rates, or to the effect of migration. The construction of a life table by Dr. Farr's shortened method (see description and formulæ on pp. 465–7) does not entail an amount of labour that should deter medical officers of health for large urban districts from undertaking it. A detailed and thoroughly practical description of the construction of an extended life table (for each year of age from 1 to 100) may be found in a Paper read before the Statistical Society in April 1883.† This description was written with a view to facilitate the operation for those whose knowledge and practice of this branch of mathematics might be insufficient to enable them to work with the sole help of Dr. Farr's formulæ, a selection from which is to be found in the following extracts.

Exigencies of space alone prompted the rejection for the purposes of this volume of a valuable contribution on what Dr. Farr called "Finance "of Life Insurance," and which he describes as "a branch of the "science of life insurance which has hitherto (in 1850) been much "neglected, and is only noticed cursorily in the standard works of Price, "Morgan, Baily, and Milne." This branch of the subject, however, is not only of a thoroughly technical character, but its interest is mainly confined to those practically engaged in the business of life insurance.

* Dr. Farr wrote, respecting these so-called healthy districts, "the sanitary condition of the people in these districts is, however, still in many respects defective."

† Journal of the Statistical Society, vol. xlvi., pp. 189–213.

This contribution may be found in the Registrar General's 12th Annual Report, pp. xvi–xxxiv.

Among Dr. Farr's other life table contributions which have been necessarily omitted from these selections through want of space, it is especially desirable to call attention to the shortened life tables for England and Wales, the Healthy Districts, London, Liverpool, and the fifteen great towns. These tables, together with others (which may be called Life Tables of Disease), showing of what diseases and at what ages a million live-born children may be expected to die in England and Wales, the Healthy Districts, Liverpool, &c., were published in that mine of statistical information and suggestion, the Supplement to the Registrar General's 35th Annual Report, in the preliminary tables, pp. lxxxiii–clxxxv.

(EDITOR.)

Death Registration, and Life and Annuity Tables.—In the Abstract of Deaths (the registration of which even for this first year has been effected with signal success) are shown the deaths of persons of each sex at every successive year of age. Such details are of acknowledged value as data for determining the laws of mortality—as bases for calculations materially affecting the interests of millions. Tables exhibiting the proportion of deaths at every successive year of age are among the most important materials from which are deduced the true principles on which should be founded the systems of life annuities and of life insurance, and the rules of Friendly Societies established for the use of the poorer classes. The materials hitherto accessible are admitted to have been too limited for framing, satisfactorily, tables to regulate the amount of contribution at various ages, by which members of such societies may become entitled to allowances in old age, or to sums payable at death. The insufficiency of the data hitherto collected, and the contradictory nature of the several tables founded on them are strongly set forth in the Report of the Select Committee of the House of Commons, in 1827, on the laws respecting Friendly Societies. It is there stated that, "according to the Northampton Tables, out of 1,000 "persons existing at the age of 25, there survive at the age of 65, 343 "persons. By the Carlisle Tables, no fewer than 513 persons will "survive;" whereby it appears "that a society which should adopt the "Northampton Tables would, if the mortality among its members should "correspond with the Carlisle Tables, have *three* annuitants where it "calculated upon *two*. Of those annuitants, moreover, a larger proportion "would live to enjoy the annuity for a considerable number of years; "for instance, of the 343 persons, who would be annuitants according to "the Northampton Tables, 98 would live for 15 years; according to the "Carlisle Tables, 162 persons would survive through that period, and "attain the age of 80 years." But still more clearly will it appear how great is the want of further facts for the elucidation of these important subjects, and the establishment of a safe standard, by viewing in a tabular form a comparison of the various results of seven approved tables of mortality (see the following table), extracted from the above-mentioned report. The recommendation of that report, that measures be adopted for making "an accurate and extensive collection of facts," whereby may be facilitated "the solution of all questions depending upon the duration "of human life," is at length carried into effect; ample materials, thus conducing to ameliorate the condition of the working classes, are now afforded in the certified copies of registers deposited in the General

Register Office; and each year's accumulation will increase the value of such records, by augmenting the number of facts upon which calculation may be brought to bear.

In pursuance of these objects, I have felt that it was of great importance not only to give an abstract for the whole kingdom of England and Wales, but to exhibit the difference which prevails in different portions of the kingdom; to compare town with country—agricultural districts with manufacturing and mining districts—the hilly with the low and level—the maritime with the inland—the eastern and northern with the western and southern parts. Nor are these diversities matters of merely curious speculation, but they may be made the source of important benefits, especially to the poorer classes. It was stated in evidence before the Committee on Parochial Registration in 1833, by the Actuary of the National Debt Office, that the extent of difference which then existed was utterly unknown—that tables for the use of the poor, in reference to sickness and mortality, and in reference to the regulation of their Friendly Societies, could not then be constructed for two districts differing in character, from the want of such information as an improved system would afford; and that,

—	By Dr. Price's Table, founded on the Register of Births and Burials at Northampton.	By the first Swedish Tables, as published by Dr. Price, for both Sexes.	By M. Deparcieux's Table, founded on the Mortality in the French Tontines, prior to 1745.	By Mr. Milne's Table, founded on Mortality observed at Carlisle.	By Mr. Griffith Davies's Table, founded on the experience of the Equitable Life Insurance Office.*	By Mr. Finlaison's Tables, founded on the experience of the Government Life Annuitants. According to his *first* Investigation, as mentioned in his Evidence in 1825.	By Mr. Finlaison's Tables, founded on the experience of the Government Life Annuitants. According to his *second* Investigation, as mentioned in his Evidence in 1827.
						Mean of both Sexes.	Mean of both Sexes.
Of 100,000 persons, aged 25, there would be alive at the age of 65	34,286	43,137	51,033	51,335	49,330	53,470	53,950
Of 100,000 persons, aged 65, there would be alive at the age of 80	28,738	23,704	29,873	31,577	37,267	38,655	37,355
Expectation of life at the age of 25 years	30·85	34·58	37·17	37·86	37·45	38·35	38·52
Expectation of life at the age of 65 years	10·88	10·10	11·25	11·79	12·35	12·81	12·50
Value of an Annuity of 1*l.* on a life aged 25, interest being at 4 per cent.	£ 15·438	£ 16·839	£ 17·420	£ 17·645	£ 17·494	£ 17·534	£ 17·634
Value of an Annuity of 1*l.* on a life aged 65, interest being at 4 per cent.	7·761	7·328	8·039	8·307	8·635	8·896	8·751
Value of a deferred Annuity of 1*l.* commencing at 65, to a life now aged 25, interest at 4 per cent.	0·55424	0·65842	0·85452	0·88823	0·88723	0·99078	0·98334

* In all the Tables above mentioned, it is to be observed that the Mortality is deduced from an equal, or nearly equal, number of each sex, with the single exception of Mr. Davies's Table, founded on the experience of the Equitable, in which office, from the practical objects of life insurance, it is evident the male sex must have composed the vast majority of lives subjected to mortality. But as it is agreed on all hands that the duration of life among females exceeds that of males, it follows that the results of Mr. Davies's Table fall materially short of what they would have been, if the facts on which he has reasoned had comprehended an equal number of each sex.

if two societies of poor men residing in districts of a totally different character were, at the same time, to apply to him for tables to guide them in preserving their societies solvent, he "should be under the "necessity of giving the same tables to both, though knowing perfectly "that the rates which were adequate in one case were inadequate in "the other." It was also stated to the Committee on Laws respecting Friendly Societies, by another eminent actuary (Mr. Milne), that no one table or scale of contributions can, with propriety, be adopted by all Friendly Societies; that one composed of members living in or near a manufacturing town required a table very different from that which would be required in places where the population is less dense, and where a considerable proportion of the members are chiefly employed in the open air; but that these are differences which he could not "pre-"tend to estimate for want of *data*." The useful principle of comparison may, if requisite, be carried out into a more minute system of subdivision than I have, in this first instance, deemed it necessary to adopt. But there was danger lest, in attempting a more subtle discrimination, we should lose sight of broad distinctions which it was important to observe; and it was necessary to remember, that to diminish by subdivision the number of facts on which calculation could be brought to bear was materially to diminish their value. The extent to which division should be carried is a question not to be decided by any established rule, and which necessarily admits of much diversity of opinion; and it has been sought to pursue a middle course between the opposite extremes of subdivision and condensation, by dividing the kingdom into the twenty-five portions in which are exhibited abstracts of deaths at different ages. In doing this, regard has been had not so much to the observance of established boundaries as to those circumstances from which diversity may be expected to arise; in some instances, contiguous counties, similar in soil, climate, elevation, and the employments of the people, have been included in the same table, while, in other instances, the boundary of the county has been disregarded where it was desirable to compare two large portions of its inhabitants pursuing very different occupations.—(1st Annual Report, pp. 15–18.)

Mortality at Groups of Ages for Life Table Purposes.—The most important use of Abstracts of Deaths is their application to the construction of tables of mortality, which, it must be remembered, are constructed, not from enumerations of deaths alone, but from two series of facts—the numbers living at different ages, and the numbers dying at the same ages—and the observed relation between those facts. This relation of the living to the dying is varying daily; but it is obvious that however complete might be the record of facts, complete beyond all conceivable possibility of attainment, these variations in the minuter portions of time would be too irregular for the safe deduction of any general laws; and that it is only by including large numbers of facts, and long portions of time, that we surmount the difficulties which such casual irregularities create, and arrive at the ascertainment of any well-founded laws of mortality.

In the assignment of these periods, the quinquennial division is found to be recommended, both by its correspondence with the enumeration we already possess of the ages of the living and by the authority of those who have already adopted it. The ages of the living in 1821 were enumerated for quinquennial periods up to the age of 20, and for decennial periods after that age. The numbers of the living at different ages were not enumerated in 1831. It is earnestly to be wished that such enumeration may be made in future, and for quinquennial periods beyond the age of 20; but it is useless to expect that an enumeration more minute than for quinquennial periods for all above childhood can be effected with success. If, therefore, the utmost to be expected with respect to the future enumeration of the living is that it be given for quinquennial periods, it becomes advisable that the age at which persons have died should be given in a corresponding manner. I may further observe, that no authentic table of mortality in practical use has ever been been calculated from an enumeration of deaths at every separate year of age; and that no actuary has yet shown that tables can be deduced more accurately from deaths so enumerated than from quinquennial or decennial periods. The well-known Carlisle Table of Mortality was calculated by Mr. Milne from Dr. Heysham's Tables of the Living and Dying, in which the ages of the latter are arranged in the same manner as in the present abstracts, except that the divisions were less minute, containing only decennial divisions after the twentieth year.

The Swedish Table was calculated by Dr. Price from abstracts of the numbers of the living and dying in Sweden during 21 years, arranged in quinquennial periods after the fifth year. The Northampton Table, the Montpellier Table, and Deparcieux's Table of Annuitants, though calculated upon data less complete than those which were the bases of the Carlisle and Swedish Tables, were formed by regulating the decrements, or by taking the mean mortality of quinquennial or decennial periods; and in addition to the eminent writers on the law of mortality above mentioned, I may quote, in support of such arrangement, Mr. Morgan and Mr. Edmonds.—(2nd Annual Report, pp. 13–14.)

History of Life Tables.—The table called by different writers *a Table of Mortality*, *a Table of Vitality*, or *a Life Table*, was invented in England by Halley the illustrious astronomer, who "first ventured "to predict the return of a comet which appeared accordingly in "1759." By this simple and elegant table the mean duration of human life, uncertain as it appears to be, and as it is with reference to individuals, can be determined with the greatest accuracy in nations, or in still smaller communities. I refer to the form, and not to the mode of construction, which has been since greatly improved.

Halley's Table was calculated on the deaths in the city of Breslau, which for various reasons he selected from the imperfect data at his disposal "as the most proper for a standard, and the rather for that the "births did a small matter exceed the funerals." He was aware that "he wanted the number of the whole people" for an accurate calculation; but Halley's Table, constructed upon nearly the same hypothesis as the Northampton Table, represented the mortality of mankind with as little inaccuracy, and was upon the whole quite as good a "standard." He observes "it may be objected that the different salubrity of places "does hinder the proposal from being universal, nor can it be denied;" "but" he concludes, "it is desired that in imitation hereof the curious "in other cities would attempt something of the same nature, *than which* "*nothing perhaps can be more useful.*" The table, which gave "a more "just idea of the state and condition of mankind than anything then "extant, had manifold uses, showing among other things the chances of "mortality at all ages, and likewise how to make certain estimate of "the value or annuity for lives, which had been previously done by an "imaginary valuation."

The Government of the Revolution, it will be recollected, introduced the system of borrowing money upon Life Annuities, and after having

failed to procure subscriptions upon the terms of the Act of 1691, succeeded in making good the deficiency by granting Life Annuities in the following year at 14 per cent. Halley, referring to the measure in his paper, remarks that his calculation shows "the great advantage of "putting money into the present fund lately granted to their Majesties "giving 14 per cent. per annum, or at the rate of 7 years' purchase for "a life, when young lives at the usual rate of interest are worth above "13 years' purchase." In the ignorance then prevailing as to the duration of life, annuities were granted at the same rate to persons of every age; and Halley pointed out "the advantage of young lives over "those in years, a life of 10 years being almost worth 13½ years' "purchase, whereas one of 36 is worth but 11."*

Tables of the lives of French annuitants, monks and nuns, were published by Deparcieux in 1746; and in 1783 Dr. Price constructed a correct Life Table from the population and deaths in Sweden and Finland. This was the first National Life Table ever made, and redounds much more to Dr. Price's fame than the Northampton Table of Mortality —so called—which, founded upon the misapplication of an hypothesis, never represented the mortality of Northampton, or of any other community, and ought not to have been published after the appearance of the admirable essay and tables of Deparcieux in 1746.†

The Carlisle Table was calculated by Mr. Milne, on two enumerations of the population of Carlisle, and its environs, made by Dr. Heysham in 1779 and 1787, with the deaths in nine years. The mean population was 8177, and the deaths 1840. Mr. Milne has described, in his treatise,‡ the care with which the observations were taken, and the method employed in the construction of this justly celebrated table, which was the first correct representation of the vitality of any portion of the English population.

"Although the *data* necessary for determining the law of mortality among the people, and the value of pecuniary interests dependent upon the continuance or failure of human life, cannot be obtained," observed Mr. Milne, in 1831, "without the active concurrence of many persons of influence and authority, yet for all the tables containing information of that kind relative to this country, and published before the year 1829, the public were indebted to the zeal and industry, and the separate efforts of a few individuals. But in March 1819, Mr. Finlaison was appointed by Government, with all the aids they could afford him, including proper assistants, and access to the registers of the nominees in tontines, and others on whose lives annuities had been granted by Government for more than a hundred years before, in which registers the exact ages at which the annuitants were nominated, and those at which they died, were stated. Thus the *data* not otherwise accessible being provided, and the labour lessened by the number of calculators employed, the expense also being defrayed by the public, at the end of 10 years, *viz.*, in March 1829, Mr. Finlaison made a report to the Lords of the Treasury, which was printed by order of the House of Commons, and in tables filling 50 folio pages, shows the rates of mortality and the values of annuities on single lives at all ages, among

* An estimate of the mortality of mankind, drawn from various tables of the births and funerals in the City of Breslau, with an attempt to ascertain the price of annuities upon lives, by Mr. E. Halley, *Transactions of Royal Society, London*, vol. xvii., 1693, p. 596, No. 196.

† Essai sur les Probabilités de la Durée de la Vie Humaine, 1746.

‡ Milne on Annuities, 1815. See also two articles by Mr. Milne in the Encyclopædia Britannica,—"Annuities" and "Mortality."

many different classes of annuitants, both separate and combined, the sexes being generally distinguished in exhibiting both the law of mortality and the value of annuities."*

The Equitable Assurance Society published in 1834 a valuable abstract of the accumulated facts in their possession, from which Mr. Morgan deduced a table of mortality. The excellent example of the Equitable Society was followed by the Amicable Society. The Societies' abstracts distinguished the persons who entered at each year of age, a point which, it is to be regretted, was neglected in Mr. Finlaison's tables, although the granting of annuities calculated on the lives of persons, sick or healthy—to selected persons in health, particularly at advanced ages, is well known to be, and has since proved, a matter of serious importance in a pecuniary point of view.

At the suggestion of Dr. Cleland, the civic authorities of Glasgow, with a laudable zeal, enumerated the ages of the population of that city in 1831; and the registration of deaths was so complete, that Mr. Milne was enabled to construct "a table of mortality, which he expects to publish," from the observations made in the 10 years 1820–30. I am not aware that any other set of observations has appeared from which a true life table can be constructed. I have already stated that Sweden is the only *nation* for which tables of this kind have been constructed upon correct principles. France has no accurate life table;† nor have the *data* from which a life table can be constructed, namely, the ages of the living and the dying, ever been published. No life tables have been constructed for the population of Prussia or of Austria; but the *data* exist, and have to a certain extent been published, though in forms which present considerable obstacles to the calculation. The Census of Prussia, in which the ages are distinguished, is taken every three years; and periodical abstracts of the deaths have been carefully made by Mr. Hoffman. The ages of the living are, however, unfortunately divided in an irregular manner, entirely different from the correct divisions adopted by Mr. Hoffman in the returns of deaths: which renders it impossible, without a preparatory interpolation, to compare the deaths with the living at the several given ages. The same objection applies to the forms of the Austrian returns. Registers of deaths are kept by the clergy of the Russian empire; but I am not aware that life tables have been framed for any portion of the Russian population. The Census has been taken decennially with great regularity in the United States of America, and the ages are properly distinguished; but abstracts of the registers of deaths have only been published by the cities of New York, Philadelphia, Boston, and some of the more advanced towns where property has accumulated, and life is watched over with more care or facility than in the back settlements —scantily peopled, with a fluctuating population. No correct life table can therefore be formed for the population of America, until they adopt in addition to the Census, the system of registration which exists in European States.

* "Annuities," Encyclopædia Britannica, 1831, p. 203.

† Duvillard states that his table, which is used by French life offices, and is given every year in the *Annuaire de France*, was founded on 100,542 deaths, at different ages, in different parts of France, among a population of 2,920,672. He has said very little about the *data*. The mean duration of life in France, according to Duvillard's table, is only 28·76 years. The duration of life is, I believe, longer in England than in any other country; but it is scarcely credible that the lives of Frenchman should be 12 years shorter than the lives of Englishmen, and 10 years shorter than the lives of Swedes. The table probably involves the same errors as the Northampton Table.

Since an English life table has now been framed from the necessary data, I venture to express a hope that the facts may be collected and abstracted, from which life tables for other nations can be constructed. A comparison of the duration of successive generations in England, France, Prussia, Austria, Russia, America, and other States, would throw much light on the physical condition of the respective populations, and suggest to scientific and benevolent individuals in every country—and to the Governments—many ways of diminishing the sufferings, and ameliorating the health and condition of the people; for the longer life of a nation denotes more than it does in an individual—a happier life—a life more exempt from sickness and infirmity—a life of greater energy and industry, of greater experience and wisdom. By these comparisons a noble national emulation might be excited: and rival nations would read of sickness diminished, deformity banished, life saved—of victories over death and the grave,—with as much enthusiasm as of victories over each other's armies in the field; and the triumph of one would not be the humiliation of the other; for in this contention none could lose territory, or honour, or blood, but all would gain strength. (5th Annual Report, pp. 16–19.)

Rural and Urban Life Tables; Surrey, London, and Liverpool.—As it might be expected, from the similarity of the human organisation, that all classes of men would, *cæteris paribus*, live on an average the same number of years, it becomes important to ascertain whether this be the case; and if it be not, to determine to what extent life is shortened in unfavourable circumstances. The life table answers this purpose; and is as indispensable in sanitary inquiries as the barometer or thermometer, and other instruments in physical research. Upon applying it in a number of well-selected cases the influence of any external cause or combination of causes can be analysed; while without its aid and extended observation and calculation we are liable to be misled at every step by vague opinions, well-concocted stories, or interested statements, in estimating the relative duration of life; which can no more be accurately made out by conjecture than the relative diameters of the sun, moon, and planets of our system.

Three examples of the application of the table to the determination of the relative duration of life in three different portions of the population of this country have been calculated; the population of Surrey (out of the Metropolis), of the Metropolis, and of Liverpool. Surrey presents a specimen of the rate at which life wastes in the country population; Liverpool is an example at the other extreme, of the effects of concentration in towns, without any adequate provision for removing the effluvia, and for securing by art the degree of purity in the dwellings and atmosphere which is partially maintained by nature in an open cultivated country. It should be distinctly understood that Surrey has not been selected as the healthiest county, and to state that it will probably be found upon inquiry that there are parts of most towns in England as unfavourable to human life as Liverpool.

The population of the extra-metropolitan parts of Surrey happens to be a little greater than the population of Liverpool, yet in 1841 the deaths in Surrey were 4,256, the deaths in Liverpool 7,556. Out of 14,450 boys under 5 years of age 2,087 died in Liverpool; of 14,045 boys in Surrey, only 699 died in the same time. By this immense mortality in Liverpool the number of males living at the age of 10–15 is reduced much below the number in Surrey at a corresponding age; the living in Surrey aged 20–30 were 18,746, but the influx of immigrants into Liverpool raised the number of males living there at that age to 23,494, who were rapidly cut down by sickness and death; so that at the age 45–55, only 7,504 males were enumerated in Liverpool, whils 9,281 were living in Surrey. From the Life Tables we shall be able to determine how many survive each successive age, and to calculate the expectation of life.

According to the Surrey observations 75,423 of 100,000 children born, attain the age of 10 years; 52,060 live to the age of 50; 28,038 to 70: in Liverpool only 48,211 of 100,000 live 10 years; 25,878 live 50 years; and 8,373 live 70 years: in the Metropolis 61,921 live 10 years; 41,309 live 50 years; and 16,344 live 70 years. The probable duration of life in Surrey is 53 years, in the Metropolis 40 years, in Liverpool 7 or 8 years: the Expectation of life does not differ so enormously; it is, however, 45 years in Surrey, 37 years in the Metropolis, and only 26 years in Liverpool; at the age of 30 the expectation of life is 35 years in Surrey, 27 years in Liverpool; at 50 the expectation of life is 21 years in Surrey, 16 years in Liverpool.

EXPECTATION OF LIFE (in Years).

Age.	Persons.			Males.			Females.		
	Surrey.	Liver-pool.	Metro-polis.	Surrey.	Liver-pool.	Metro-polis.	Surrey.	Liver-pool.	Metro-polis.
0	45	26	37	44	25	35	46	27	38
1	50	33	43	50	33	41	50	34	44
2	51	38	46	51	37	45	52	39	48
3	52	41	47	52	40	46	52	42	49
4	52	42	48	52	41	46	52	43	50
5	52	43	48	51	42	46	52	43	50
10	49	41	45	49	41	44	49	42	47
15	45	37	41	45	37	40	45	38	43
20	42	34	38	42	33	36	42	34	39
25	38	30	34	38	30	32	38	31	35
30	35	27	30	35	27	29	35	27	32
35	31	24	27	31	23	25	31	24	28
40	28	21	24	28	21	22	28	22	25
45	24	18	20	24	18	19	24	18	22
50	21	16	18	21	16	17	21	17	18

It might be cited as an illustration of the necessity of registration and of calculation in these matters, that, before the Annual Abstracts of Deaths were published, some of the best informed people believed Liverpool one of the healthiest spots in England; and the late Mr. Rickman inserted, doubtless on what he at the time considered good authority, the following note in the Population Abstracts of 1831:—"The great *increase in the town of Liverpool is attributed to the* "*salubrity of the air*, and the progressive improvement in its trade, "commerce, steam navigation, and railroads."

It has been stated that the mean duration of life in Surrey is about 45, in Liverpool about 26 years; now if all the inhabitants lived 45 years in Surrey and 26 years in Liverpool, the difference would be obvious; but such is not the law of nature; in both a certain number of deaths takes place at all ages, and at the Census 3 males and 11 females were returned as living in Liverpool at the advanced age of 95 years and upwards. Little dependence, it is true, can be placed upon the statements of age in the table deduced from the returns in one year

(1841) after the age of 90; but though it is quite possible that isolated individuals may live 100 years in Liverpool, they have little or no effect on the average duration of life, which differs from that of Surrey, as has been already seen, in the proportion of 26 to 45 years.—(5th Annual Report, pp. 31–7.)

Uncertainty of Individual Life and Constancy of Averages.—Addison, in one of his popular papers, "The Vision of Mirza," has an allegory which was probably suggested by Halley's table; he compares "human life to a bridge consisting of threescore and ten entire arches, "with several broken arches, which, added to those which were entire, "made up the number to about a hundred." "I see multitudes of "people passing over it," said I, "and a black cloud hanging on each "end of it. As I looked more attentively, I saw several of the "passengers dropping through the bridge into the great tide that "flowed underneath it; and upon further examination perceived there "were innumerable trap-doors that lay concealed in the bridge, which "the passengers no sooner trod upon, than they fell through into the "tide, and immediately disappeared. These hidden pitfalls were *set* "*very thick at the entrance of the bridge*, so that throngs of people "no sooner broke through the cloud, but many of them fell into them. "They *grew thinner towards the middle, but multiplied and lay closer* "*together towards the end* of the arches that were entire." Our life table follows "a throng" of 100,000 that "brake through the cloud" into life at the same moment, and counts them as they step on every arch. It shows, therefore, how many fall through the "hidden pitfalls." The danger is exactly measured. The arches over which sickly multitudes pass, are the same in number as those traversed by a healthy people; but the "trap-doors" and "hidden pitfalls" in their way are twice as numerous, though they can only be perceived by careful observation and counting; while a difference of 26 and 45 "arches" would be obvious to the unassisted eye.

In the law which regulates the waste of life two things have been reconciled: the uncertainty of the hour of death, and the constancy in the same circumstances of the mean duration of man's existence. The days of successive generations are numbered, yet a child born to day may die in any day, hour, or minute, of the next *hundred years;* and until a very advanced age the chances always are that the time of death will be several years distant: the danger of death we know varies at different ages, and in different states of health; but if the limit of life be 100 years, it is on an average 36,525 to one that a person will not die on a given day; 876,600 to one that he will not die on a given hour, and 52,596,200 to one that he will not die at a given minute. These chances—which vary as life advances—are so low that practically they have little or no influence in ordinary affairs; and as a general rule men have no fear of dying upon any *day;* yet the knowledge that they *may* die at any instant exercises a salutary check upon their conduct; and, notwithstanding its sometimes appalling effects, the changing certainty or uncertainty of life, according to the different aspects and points of view, is in harmony with the feelings, hopes, moral constitution, and destinies of mankind.

The serious disadvantage which arose from the difficulty of perceiving the changes in the duration of life, and consequently the influence of external causes upon health and longevity, has now been overcome in this country by the arduous labours of scientific inquirers, and by the conjoint enumeration of the ages of the population and the registration of births and deaths.

Man does not pass through all the stages of his physiological and intellectual development in less than 70 years: yet it has been shown that in the most favourable circumstances in which large bodies of the English population are placed, the mean life attained is only 45 years; and that other large bodies of the people fall short of this relatively low standard, to the extent of nineteen years—years of childhood and youth principally—years of toil too and poverty perhaps, but of life—years also of manhood in its prime, wisdom in its maturity, virtue in its height of usefulness and glory. The facts and calculations upon which these results rest, will not be disputed by those who have studied the subject most deeply; I believe that they will be confirmed by the still more extended data which are every year accumulating under the present system of Registration. In the mean time enough has been advanced to direct public attention to the "hidden pitfalls," which had so long lain concealed, which destroy every year thousands of lives, and which it is believed admit, to a considerable extent, of removal by the judicious application of sanitary measures.—(5th Annual Report, pp. 37-8.)

Mean Duration of Life and Mean Age at Death.—As the mean duration of life, technically called the expectation of life, differs very widely from the "mean age at death," and from some estimates which have been made of the relative health of different portions of the population, it may be right, before I close this Report, to point out the errors into which inquirers are liable to fall in reasoning upon the "age at death;" or, which is the same thing, constructing life tables from the deaths alone. Mr. Milne has very clearly pointed out the fallacies of all calculations and pretended tables of mortality, founded upon returns of the ages at death alone; and I should consider it sufficient to refer to his able article "Mortality" in the "Encyclopædia Britannica," if the error had not survived and assumed new forms very much calculated to mislead those who have had time to pay but a cursory attention to the subject.

The duration of life in England is 41 years; if the population were *stationary* the mean age of those who died would be 41 years; and 1 in 41 would die every year. The population has however increased 1·41 per cent. annually during the last 40 years; and we find that the mean age of the persons who died in the year 1841, instead of being 41, is 29 years; while 1 in 46 of the population died. This agrees with what Mr. Milne lays down as the result of other observations, that "when the population has been increasing, the mean duration of life "according to the table will be less than the number out of which one "person dies annually in that population, but *the difference will be* "*small* except under particular circumstances;" and again, that the mean age at which persons die "will fall short of the number of the people "out of which one dies annually *by a much greater number* than in "the case we have just been considering." "When the proportion of "the people dying annually is known," he adds, "it will not be "difficult to judge whether a table of mortality for that people has "been constructed properly from the necessary data; or, what is much "more common and more easily effected, by summation of the deaths "at all ages."* "The mean age at death," it may be here stated, is obtained by simply summing up the ages at which people die, and

* By applying this test, Mr. Milne has shown that some recent life tables—the mode of constructing which has not been explained—have been constructed from the ages at death alone—apparently without any kind of correction.—*See* Article "Mortality" before referred to.

dividing the number of years by the number of deaths. It is only a pity that the method is not as accurate as it is easy.

Its errors may be further illustrated by comparing the English with two foreign observations:

	Mean duration of Life.	Mean Age at Death.	One Death in
England, (1841) -	41 years.	29 years.	46 Living.
France, (1817-31) -	40 "	34 "	42 "
Sweden, (1801-5) -	39 "	31 "	41 "

The average age of the persons who died, or the "mean age at death," was 34 years in France, 31 years in Sweden, 29 years in England; yet we know that the "expectation of life" is greater in England than in Sweden or in France. A Society that granted life annuities to children in England would have to make 40 annual payments on an average, and only 38 in Sweden. The annual funerals were inversely as the "mean "age at death," or 1 in 41 in Sweden; 1 in 42 in France; 1 in 46 in England. Mr. Milne has stated the fact, at first sight paradoxical, that in an increasing population the average age at death is less, and the annual mortality less, than in a stationary population having the same expectation of life. I will endeavour to explain the cause of this as briefly as possible.

The births exceed the deaths in England, and in the year 1841 the births registered amounted to 512,158, the deaths to 343,847. If the population were stationary the births would be 343,847; they would maintain the existing population; but the annual excess of 168,311 children, more or less, which have been thrown for many years into the English population has produced a preponderance of the youthful over the aged part of the population. If the law of mortality had remained constant, and the births and deaths had been equal for the last century, it would have been found that on an average about 35 in 100 of the people were under 20, and 14 in 100 above 60 years of age; but it appears from the last Census that 46 in 100 were under under 20, and only 7 in 100 above 60 years of age.

The people are younger than in France, or Sweden; the mean age obtained by dividing the sum of the ages of those who die in England by the number of deaths is consequently lower than the age at death in Sweden and France. But why, it may be asked, is the mortality 1 in 46 if the expectation of life be 41 years? The reason is, that as the increase of the population has been long and progressive, an excess has been accumulated of persons between the ages of 5 and 55, among whom the mortality is lower than it is among persons of all ages. With the reduction in the relative numbers above the age of 60 this has more than compensated for the high rate of mortality among the excessive number of children under 3 years of age; and has reduced the mortality below 1 in 41 annually, which it would be if the population were stationary. As the populations of France and Sweden have not increased more than half as fast as the English population, the diminution of the age at death has been less considerable, though sufficient to derange all calculations and all comparisons, such as that of the "mean age at death" deduced upon the supposition that in the populations compared the births and deaths have been equal,—the mortality uniform,—for a long series of years.

The deaths of children under 1 year of age were 74,210 in the year 1841, and the total deaths 343,847. But it must not be inferred in cases of this kind, as it has been frequently, and as it always is in tables of mortality deduced from the ages at death alone, that 74,210 of 343,847 children die in their first year. Nothing can be more erroneous: the deaths occurred out of a number certainly not less, and probably more, than 512,000; for though all the births have not been registered, the births of 512,158 children were registered in the year 1841, and 502,303 in 1840. The error is as striking when the deaths under 5 years of age are compared with the total deaths, instead of the births, in the preceding years.

If the reasoning upon "the mean age at death" be employed to determine the relative salubrity of towns and professions as well as of different classes of the community, the nature of the results may be readily divined. The mean age at death is 29 years in England, 29 years in the Metropolis, 34 years in Surrey; the true mean durations of life being nearly 41, 37, and 45 years, so that the errors by this method amount to 12 years, 8 years, and 11 years! The rate of increase, the duration of the increase of population, the emigration, the relative numbers of children and adults, the mean age of the living—upon all of which the "mean age at death" depends—differ in town and in country, in agricultural and manufacturing districts, to an extent which renders any application of the method to the construction of local life tables, or to the calculation of the relative duration of life, difficult and doubtful, if the proper corrections be made; absurd and misleading, if the "mean age at death" be taken to represent the expectation of life.

The numbers following different professions fluctuate more than the general population; the relative proportion of young and aged persons varies from year to year; certain professions, stations, and ranks are only attained by persons advanced in years; and some occupations are only followed in youth; hence it requires no great amount of sagacity to perceive that "the mean age at death," or the age at which the greatest number of deaths occurs, cannot be depended upon in investigating the influence of occupation, rank, and profession upon health and longevity. If it were found, upon an inquiry into the health of the officers of the army on full pay, that "the mean age at death" of "Cornets, Ensigns, and Second Lieutenants" was 22 years; of "Lieu- "tenants" 29 years; of "Captains" 37 years; of "Majors" 44 years; of "Lieutenant-Colonels" 48 years; of general Officers, ages still further advanced—and that the ages of Curates, Rectors, and Bishops; of Barristers of seven years' standing, leading Counsel and venerable Judges—differed to an equal or greater extent, a strong case may no doubt be made out on behalf of those young, but early-dying Cornets, Curates, and Juvenile Barristers, whose "mean age at death" was under 30! It would be almost necessary to make them Generals, Bishops, and Judges—for the sake of their health. The Assurance Societies are happily so considerate and liberal that they do not attach the slightest importance to the mean age at death, but assure the lives of young men of all the professions at the age of 24 upon the assumption that they will live 38 or at the least 31 years, and pay 38 or 31 annual premiums on an average before they die; while they make the Bishops, Judges, and Generals who go to insure their lives at 60 pay as if they would live but 13 or 14 years.

It has been somewhere stated that the "mean age at death" of dress-makers is exceedingly low, and this has been adduced as a proof of the destructive effects of their employment. If the inquiries had been extended to boarding schools, or to the boys at Christ's Hospital, the "mean age at death" would have been found still lower. Mr. Grainger states, in his interesting Report, that the majority of dress-makers are between the ages of 16 and 26; and it is understood that if they die

after they marry they are not often designated by that title in the register. This source of error and the increase of population will be found to affect the estimate of the influence of other occupations. That the lives of dress-makers are very much shortened by the severe hardships and ignorant mistreatment to which they are exposed cannot be doubted; but false arguments injure instead of aiding their cause.

In a thriving commercial country like England there is a general movement, such as has been noticed in the army and the liberal professions—from the lower into the higher ranks of society. The servant becomes a master; shop-boys grow into merchants or aldermen; the tradesman retires and is classed either as "independent," "in easy "circumstances," or a "gentleman," at the Census, and in the mortuary registers. But these promotions as a general rule are slow; and those only attain the higher positions who live long. If the mean age, at which masters and servants, the wealthy and indigent die, were noted and made the basis of any reasoning respecting the relative health and longevity of the lower and upper classes, the differences would evidently be exaggerated. The exaggeration is increased in another way; many poor people are reduced to seek an asylum at advanced ages in the workhouses, and are not often designated by the occupations which they followed in manhood, but by the general name "paupers": the ages of those who die in the ranks of their respective trades and professions are thus reduced to the same extent as the ages of the paupers who die in workhouses are raised above the average. In 1841 the mean age of 45,507 persons who died in London was 29 years; the mortality was 1 in 40; in the same year 4282 persons died in the London workhouses at the advanced age of 49 years, which they must have nearly attained before they entered those establishments, inasmuch as the mortality there appears to have been about 22 per cent., or 1 in 5 annually.* Contrast 49, the "mean age at death," of paupers in the workhouses, with other statements, which make the "mean age at "death" of the same or a superior class of persons 16 or 20 years.

One in 116 of the boys in Christ's Hospital died annually, in the 12 years 1831–42; the mean age of the boys who died was 11 years. The "mean age at death" and the mortality were both low. This illustration, taken from an extreme instance, shows why, while the mortality is lower, the mean age at death is less in England than in some other countries. The English population contains more young persons, more of the age of the Christ's Hospital boys, than the foreign populations.

The life table affords the most satisfactory measure of the relative duration of life, either of classes or of different communities. The mortality obtained by dividing the deaths by the living at each age, is also an unimpeachable test; it is the preliminary to the construction of a true life table. The ratio of the total deaths to the total population affords the next best test that can be employed; if the populations compared be of the same age, their relative mortality will be correctly given by this method; if the ages and the rate of increase differ, the "mean duration of life will be less than the number out of which *one* "dies annually, but the difference will be small in the increasing popu- "lation." The "mean age at death," or pretended life tables constructed from the deaths, without reference to the ages of the living, or the ages of the living without the ages of the dying, are, as I have already stated, only calculated to mislead in inquiries of this kind, unless great care and discrimination be employed in their application. It happens, nevertheless, in some cases that they afford the only resource; the total

* The pauper population of the London Workhouses was 19,412 at the time the Census was taken in June; it would probably be greater in Winter.

deaths are not registered, or the ages of the living have never been enumerated; the ages at death may then be compared with the ages at death of *other populations—known to be increasing at nearly the same rate*, or corrections may be made upon the hypothesis of a uniform rate of increase during a certain number of years.

In France, for instance, where the ages of the living have never been abstracted, M. Demonferrand has constructed a life table, which is probably not very erroneous, from the ages at death; assisted, however, most essentially by the complete registration of births, and the annual enumeration of the young men who attain the age of 20. In Ireland, where the deaths are not registered, abstracts have been made of the ages of the population; and the Commissioners of the last Irish Census included in "the personal return a table in which every head of a "family was requested to insert all the deaths which had occurred in "his family within the last ten years, stating the cause of each death, "and the age and occupation of the deceased." In addition to this, they sent to every hospital and lunatic asylum forms which were filled up. From the ages at death thus obtained they constructed tables, which will be found in pages 80–82 of their Report—a Report, I may remark in passing, which is replete with valuable information. As the population of Ireland, notwithstanding the emigration going on, increased 14·19 per cent. in the 10 years ending 1831, and 5·25 per cent. in the 10 years ending in 1841, the births must exceed the deaths. The tables, which could only be correct if the births and deaths had been equal, and there had been no emigration, are therefore not true life tables; and the "expectation of life" deduced from them, which is 29 years at birth in the "rural," and 24 years in the "civic" districts, must be understated to a certain but unknown extent. The deaths returned for the year 1840 were 141,536, making the mortality 1 in 57·5: the return is evidently defective, but, as the Commissioners correctly remark, this will not affect "the mean age at death," provided that the deaths omitted occurred at the same ages as those returned. The Commissioners had not at their disposal the *data* requisite for calculating the true "expectation of life" in Ireland: the application of the term is an inadvertency rather than an error, and I should not have noticed it here, if the tables had not afforded some interesting points of comparison with the English observations. Tables have been constructed from the deaths in England, upon the same plan as the Irish tables; and if the population of the two Divisions of the United Kingdom had increased for many years at nearly the same rate, or if the emigration had been nearly in the same proportion, the "expectations of life," as we may call them for a moment, will be equally erroneous: they may therefore be compared. The results are given in the following tables.

Expectation of Life.—Males and Females.
(Erroneously deduced from the Deaths alone.)

Age.	Males.			Females.		
	England.	Ireland, Civic Districts.	Ireland, Rural Districts.	England.	Ireland, Civic Districts.	Ireland, Rural Districts.
0	28	24	30	31	24	29
5	42	35	41	42	35	40
20	35	28	34	36	29	33
40	25	19	23	27	20	23
60	14	11	13	15	12	12

Expectation of Life.—Persons.
(Erroneously deduced from the Deaths alone.)

Age.	Ireland.		England.					
	Civic Districts.	Rural Districts.	England and Wales.	Surrey.	Metropolis.	Liverpool.	London,* 1728-37.	London,† 1759-68.
0	24	29	29	34	29	21‡	19	26
5	35	41	42	44	42	36	36	39
20	28	33	35	37	33	28	29	29
40	19	23	26	27	22	19	20	20
60	11	13	14	15	12	11	12	12

* From Simpson's Select Exercises. The observations were corrected (?) by Simpson.
† Deduced from Dr. Price's Table 13.
‡ The more exact "mean age at death" in Liverpool is 20·54.

The "expectations of life" (if we may call them so,) are the same at birth in the "rural districts" of Ireland as in all England; they are less after 20, but agree remarkably at all ages with the expectation of life in the Metropolis. This resemblance between a table of the ages at death in a city, and the ages at death in the rural districts of Ireland, would be caused to a certain extent by a diminution in the latter of the proportion of births within the last 10 years. The (erroneous) "expectations "of life," in the "civic" districts agree after 20 with those of Liverpool; at birth the expectation is higher in the Irish towns than in Liverpool, but lower than in London. The "expectation of life" therefore is not so low as it is represented in the tables of the Commissioners. Judging from the analogy of the English tables, the expectation of life is less in Ireland than in England; the inhabitants of the "civic" districts, perhaps, attaining a mean age of 30 years, while in the rural districts they live on an average to 37. The life tables of the Metropolis, and Liverpool or Manchester, would probably apply to the two sections of the Irish population; but this can of course be only conjecture. The Irish tables may be corrected to a considerable extent by means of the ascertained rate of increase, and the enumerated ages of the living; some of the methods to be employed were investigated by Euler, and are given by Lacroix in his "Traité élémentaire des Probabilités,"* p. 207. Implicit confidence, however, could not be placed in the results.

It is a curious feature of the Irish tables that the men appear to live longer than the women in "rural" districts; and the women longer than the men in "civic" districts. Frenchmen live longer after 20 than the women, if the expectations of life in the two sexes be equally correct or incorrect in M. Demonferrand's tables. In England the lives of females exceed those of males by about a year—except at birth, when the difference is greater. In Surrey the females from the age of one year and upwards live little longer than the males; the difference is greater in the Metropolis, where it amounts, at some ages, to two or three years. This may, perhaps, account for the differences in the expectations of life deduced from male and female annuitants. According to Mr. Finlaison's tables—the lives of men are from four to six years shorter than those of women; a discrepancy which in its extent is entirely at variance with all other observations. If the majority of the annuitants before 1829 were inhabitants of London, and more than a

* Mémoires de l'Académie de Berlin, anno 1760, p. 144.

due proportion of the women lived in the country, such a discordant result would however be produced.—(5th Annual Report, pp. 38–46.)

Mean Duration of Life deduced from incomplete Observations.—A publication has appeared professing to give the mortality which has prevailed among seventeen life offices.* The author, Mr. Jones, states that "by the liberality of several of the life offices, and the disinterested "zeal and services of a committee of some of the most experienced "and eminent of the actuaries, we have now data for the construction "of a rate of mortality, not simply of the experience of the Equitable "and Amicable, but of the combined experience of no less than 17 life "offices, embracing 83,905 policies; and a rate of mortality has been "adjusted by one of the most eminent mathematicians on the committee, "from the combined town and country experience, embracing 62,537 "assurances."—(*Int.*, p. x.) "The committee state that "the most "striking features exhibited in these tables are the *great mortality* "*that prevails among Irish lives, and the marked difference in the* "*rate of mortality among males and females.* The near agreement with "each other of the tables for 'town' and 'country' assurances is also "very remarkable, considering that no adjustment has been employed." "(p. xvi.) The committee very justly observe that their tables "represent a *lower rate of mortality than can be expected to prevail* "*in a longer period of time than that over which the present observa-* "*tions extend;* for the average duration of policies embraced in nearly "one-half of the experience is *under* $5\frac{1}{2}$ *years;* and taking the whole "of the experience together, which includes that of the 'Equitable' "and 'Amicable,' the two oldest offices existing, the average duration "of all the *policies* is *not* $8\frac{1}{2}$ *years.*" (p. xix.)

These tables are exceedingly interesting, as they show the experience of the life offices so far as it extends, and the actual effect of their more or less imperfect selection of lives. It is an objection to all tables framed in this manner, on the experience of life offices and on annuitants, that you have to wait 50 or 100 years before all the lives have expired, and have then, in applying them practically, to assume that the future annuitants, &c., will be selected on the same principles, and be placed in the same circumstances.

The most conflicting results are necessarily obtained by the incomplete observations; thus, while Mr. Finlaison's Table makes females at 20 live 44·0 years, and males 38·4 years, the Actuaries' Table presents a result exactly the reverse: females selected for assurance at 20 have, according to their table, an expectation of 35·9 years, males of 39·8 years!

It will be observed that the expectation of life among males, by the English Table, lies between the expectations of life for males by Mr. Finlaison's and the Actuaries' Tables; it agrees very closely with the mean expectation of the two tables. The expectation by the former, at the age of 41, is 26·39 years, by the latter 25·42; the mean is 25·91 years; and the expectation of life is 25·91 by the English Table; so it happens that neither of the tables from incomplete observations is very incorrect for males. With regard to the expectations of females, Mr. Finlaison's and the actuaries' statements differ to the extent of eight years at the age of 20; at the age of 26 the expectations, according to the two statements, are 40·17 years and 33·79 years —difference, 6·38 years—mean, 36·98 years. The expectation of life for a woman aged 26 is 36·86 years by the English Table; Mr. Finlaison's result is 3·19 years above, the Actuaries' 3·07 years below the

* A Series of Tables, &c., by Jenkin Jones, 1843.

average of the national table; while the expectation of life for males is nearly the same at 26 by the three tables—namely, 35·41, 35·88, and 35·65 years.

DIFFERENCES from the English Table, by excess or defect, in the Expectations of Life deduced from Mr. Finlaison's and the Actuaries' Tables.

Age.	Males.			Females.		
	Expectation of Life by English Table.	Differences by Mr. Finlaison's Table.	Differences by Actuaries' Table.	Differences by Actuaries' Table.	Differences by Mr. Finlaison's Table.	Expectation of Life by English Table.
	Years.	Year.	Year.	Defect in Years.	Excess in Years.	Years.
20	39·88	−·49	−·04	−4·95	+3·18	40·81
25	36·47	−·57	+·16	−3·11	+3·29	37·52
30	33·13	+·04	+·04	−2·52	+3·32	34·25
35	29·83	+·34	−·31	−1·92	+3·32	30·99
40	26·56	+·46	−·50	−1·36	+3·40	27·72
45	23·30	+·45	−·67	−1·22	+3·38	24·43
50	20·02	+·28	−·61	−1·02	+3·28	21·07
55	16·68	+·47	−·47	−0·85	+3·16	17·63
60	13·59	+·80	−·12	−0·62	+2·92	14·40
65	10·86	+·77	+·01	−0·92	+2·48	11·52
70	8·51	+·71	−·17	−1·10	+1·96	9·03
75	6·53	+·59	−·50	−1·55	+1·54	6·92
80	4·92	+·02	−·17	−0·45	+1·30	5·20

Thus, at the age of 40, the mean future duration of life is 26·56 years for males, according to the English Table; or ·46 of a year more by Mr. Finlaison's Table; and 0·50 years less by the Actuaries' Table: at the same age for females, the Actuaries' Table differs from the English Table 1·36 year by defect; Mr. Finlaison's Table 3·40 years by excess. The sign +, *plus*, denotes excess over the expectation by the English Table. The sign —, *minus*, denotes the reverse.

The expectation of life is less for *males* by Mr. Finlaison's than by the Actuaries' Table up to the age of 30: it is afterwards more; but in females the difference in the expectation is enormous.

Age.	Difference in the Expectation of Life by Mr. Finlaison's and the Actuaries' Tables.	
	Males.	Females.
	Years.	Years.
20	− ·45	+8·13
25	− ·73	+6·40
30	·00	+5·84
35	+ ·65	+5·24
40	+ ·96	+4·76
45	+1·12	+4·60
50	+ ·89	+4·30
55	+ ·94	+4·01
60	+ ·92	+3·54
65	+ ·76	+3·40
70	+ ·88	+3·06
75	+1·09	+3·09
80	+ ·19	+1·75

The sign + denotes excess of the expectation by Mr. Finlaison's over that by the Actuaries' Table.

Although the tables of the "Actuaries" and their remarks, referred to in this publication are, from some cause not satisfactorily explained, inaccessible to the public, it would appear from the statements of Mr. Jones that the tables are considered by them incomplete, and not a safe basis for the guidance of pecuniary transactions. It is scarcely necessary to add that, although no such great difference in the longevity of the two sexes exists in nature, it can readily be conceived that men and women may be selected whose lives differ, or appear from an incomplete series of observations to differ, as much as the lives of the males and females in the Actuaries' and Mr. Finlaison's Tables. (5th Annual Report, pp. 338–41.)

Construction of Life Tables; De Moivre's Hypothesis.—De Moivre gave his name to an hypothesis, according to which the numbers living decrease in an arithmetical progression down to nothing, at the age 86.* It has been since assumed, as stated by Mr. Milne, that "*the* " *number* of the living in any year of their age is an arithmetical mean " proportional between the numbers that annually enter upon and that " annually complete that year."† If δ deaths occur in a year, upon this hypothesis, they are assumed to take place "at δ equal intervals"; and it is by the same hypothesis that, in calculating the expectation of life, writers assume that "the number of living of the age of n years " and upwards is less than the sum of those that annually complete " that and all the greater ages by half the number that annually com- " plete that year of their age."—(*Milne*, pp. 85–6.) This hypothesis, which is interwoven into all the calculations of interest and of life annuities, brings them within the range of algebra; for, without the assumption that the interest of money and the mortality remained uniform for some certain definite time, the resources of the calculus must be called into requisition. The errors which result in life assurance from the hypothesis of an equal decrement are small and quite insignificant when compared with the errors of observation, and the errors incurred by the assumption that the interest of money and the mortality will remain stationary for a long series of years. Still, it must be borne in mind that the rate of mortality varies (insensibly) every moment, and that the errors involved in the hypothesis are greatest in the first year of life. By making the births the basis of the table (if the births are all registered), the decrement in the first year, where the error would, by the other method, be of some magnitude, will be correctly represented. The deaths in the second year of life, out of 100 constantly living, were 6·503; and, by the hypothesis, 1·03252 would be alive at the beginning, ·96748 at the end of the second year; the fraction $\frac{96748}{103252}$ would therefore express the chance of living the second year. If 43,104 were alive at the beginning, 40,388 would be alive at the end of the second year; for 103252 : 96748 : : 43104 : 40388; or $\frac{96748}{103252} \times 43104 = 40388$. In this manner the series, down to 5 years may be calculated. The mortality against the age 5–10, namely, ·00955, was taken to represent the mortality of the middle year (in this instance 7–8), and the mortality of the intermediate years was interpolated. As the series is short, and terminates at 10–15, it is not easy to test any theory of interpolation, particularly as the mortality at 10–15 is, I believe, through the error of speaking in tens, understated. It is improbable that the mortality of 10–15 should be 50 per cent. lower than the mortality at 15–20. Neither this nor any other table which I have seen, derived directly from observation, is very satisfactory up to the age of 15; although the earlier ages must generally be known, they have not been so correctly stated at the censuses as could be desired. Whatever system of interpolation may be employed, however, the expectation of life will not be much affected by it from 1 to 15; and the numbers after 15 are quite independent of those before 15. In framing the English Table (No. 1), the *mortality* at every age was interpolated by the logarithm which expressed the ratio of the increase in the mortality at every year of life; and the chance of living each year was deduced from $\frac{1-\frac{1}{2}m}{1+\frac{1}{2}m}$.

* Treatise of Annuities on Lives; Preface, &c., by De Moivre.

† Milne on Annuities and Life Assurances, p. 85.

I do not find that Dr. Price ever explained the method of interpolation which he employed in framing the Northampton or Swedish Tables of Mortality. It was probably empirical. Mr. Milne has adduced, in his excellent Treatise, a method by which he says, "when " the number of the living and of the annual deaths are taken for " intervals of several years each, the number of the living in each " particular year of their age, included in any one of those intervals, " may be interpolated with sufficient exactness."* He has also given, in the eighteenth table of his work, "the logarithm of the fraction " which measures the probability that a life of an assigned age will " survive one year, according to the *Carlisle* Table of Mortality." (5th Annual Report, pp. 349–50.)

A Short Method of constructing Life Tables.—The arithmetical labour involved in the construction of correct life tables, showing the living at every year of age, is very considerable. But for a great many purposes the number surviving every five years, after the five first, and the expectations of life at those intervals, furnish quite sufficient information. These results were obtained by employing the following method in calculating the life tables for the Metropolis, Surrey, and Liverpool:—

Up to the age of five years the method is the same as that already described,† and it was thus found that of 50,521 boys born in Surrey, 43,637 live a year, 41,857 two years, 40,704 three years, 40,031 four years, 39,550 five years. The next point was to determine how many of the 39,550 attain the age of 10 years. The living enumerated at the age 5–10 were 13,588, the deaths 145; and after the proper correction the mortality m was ascertained to be ·01050; so $\frac{m}{2}$ = ·00525, and $\frac{1-\frac{1}{2}m}{1+\frac{1}{2}m} = \frac{\cdot 99475}{1\cdot 00525} = \cdot 98955$ the probability of living *one year* at the middle of the period, or at seven and a half years of age. But it may be assumed that $\left(\frac{1-\frac{1}{2}m}{1+\frac{1}{2}m}\right)^5 = p_{5,5}$ = the probability of living the *five years* from the age of 5 to 10; and $(\cdot 98955)^5 = \cdot 94885$; which, multiplied by 39550, gives 37527 = the numbers surviving at the age of 10.

* Annuities and Assurances, p. 100.

† *See* previous extract, p. 464.

If the calculation be continued down to 15, 20, 25, and every fifth year to the end, the following table will be obtained :—

SURREY LIFE TABLE—Males (1841).

Age.	Living.	Quinquennial Periods + $\frac{1}{2}lm$.	Age.	Living.	Quinquennial Periods + $\frac{1}{2}lm$.
0	50,521	476,444	40	29,822	179,047
			45	28,069	149,225
1	43,637		50	25,973	121,156
2	41,857		55	23,892	95,183
3	40,704		60	21,459	
4	40,031		65	18,235	
			70	13,976	
5	39,550	425,923	75	9,836	
10	37,527	386,373	80	5,393	
15	36,469	348,846	85	2,031	
20	35,338	312,377	90	290	
25	34,061	277,039	95	58	
30	32,742	242,978	100	11	
35	31,189	210,236	105	2	

Add up the column headed "living" to the number 39,550 (against the age 5 years), and the sum will be the number of five years—of *lustres*—which the 39,550 persons will live + $\frac{39,550}{2} = 19,775$. Subtract, therefore, 19,775 from the sum 425,923, and 406,148 will remain; which, divided by 39,550, gives for quotient 10·269 *lustres* as the expectation of life at that age. A lustre is five years; consequently the expectation of life in years is five times 10·269, or 51·3 years. If 425,923 be divided by 39,550, the quotient will be 10·769; and 10·769 — ·5 = 10·269, the same result as before. The expectation of life will be found to be 34·5 years at the age of 30.

The number of living at every five years except the first, deduced by this method, may be considered nearly correct; the expectation of life is slightly overstated by the assumption that the living at the ages 5, 6, 7, 8, 9, 10; and 10, 11, &c., are series in arithmetical progression. The error does not exceed one-tenth part of a year from 5 to 60 years of age. At birth, and after 70, it does not exceed half a year, which may be subtracted as a correction. But by calculating the number surviving every year up to the age of five, a sufficiently close approximation to the expectation of life at birth will be obtained. The years of life under five are $\frac{5}{6} \times 256,300 = 213,583$; and the years of life, after the age of five $= 5 \times (425,923 - 19,775) = 2,030,740$, and $\frac{2,030,740 + 213,583}{50,521} = 44 \cdot 4$, a boy's expectation of life at birth in Surrey.

A life table still shorter may be constructed by taking intervals of 10 years, and using $\left(\frac{1 - \frac{1}{2}m}{1 + \frac{1}{2}m}\right)^{10}$. The errors in the calculation of the expectation of life from the living at every tenth year, can be corrected. They are always of the same nature. If we take the numbers "living" against every 10th year from the English Table, it will be found that the excess of the expectations of life, ranges at the ages 10 to 50, from ·1 to ·2 or ·3 of a year. At birth the true expectation will be obtained very nearly by subtracting one year from the expectation, derived from the decennial table.

By adding up the column headed "living," in the subjoined table, dividing by the first number 100,000, multiplying by 10, and sub-

tracting 5, we obtain 42·05 years as the expectation of life, which is too much by nine-tenths of a year.

Years.

Age 0 $\frac{470,530}{100,000} \times 10 = 47 \cdot 05$; and $47 \cdot 05 - 5 = 42 \cdot 05$

True expectation of life 41·16

Error ·89

Age 10 $\frac{370,530}{70,612} \times 10 = 52 \cdot 47$; and $52 \cdot 47 - 5 = 47 \cdot 47$

True expectation of life 47·44

Error ·03

DECENNIAL LIFE TABLE.—(From the English Table.)

Years.	Living.	Expectation of Life.
0	100,000	42·05 — ·89 = 41·16
10	70,612	47·47 — ·03 = 47·44
20	66,059	40·40 — ·06 = 40·34
30	60,332	33·76 — ·08 = 33·68
40	53,825	27·23 — ·09 = 27·14
50	46,621	20·67 — ·12 = 20·55
60	37,996	14·23 — ·23 = 14·00
70	24,531	9·29 — ·51 = 8·78
80	9,398	|| || ||
90	1,140	By the decennial table — Error = By the annual table
100	16	

(5th Annual Report, pp. 362–5.)

Mean Duration of Life in Metropolitan Districts.—When sufficient data have been collected, it is proposed to calculate the mean duration of life, or the expectation of life, for different parts of the metropolis. Several corrections have to be made. The following is a specimen of a Decennial Life Table for two districts. It was computed and corrected (in the manner already described)* on the population and deaths of 1841,

ST. GEORGE, HANOVER-SQUARE.

Decennial Life Table.				Expectation of Life.	
Age.	Persons.	Males.	Females.	Males.	Females.
0	100,000	51,949	48,051	37·4	39·7
10	63,732	33,011	30,721	47·0	50·2
20	60,434	31,176	29,258	39·5	42·5
30	57,178	29,016	28,162	32·1	34·0
40	52,266	26,097	26,169	25·2	26·2
50	45,451	22,279	23,172	18·7	19·0
60	36,048	17,926	18,122	12·2	13·1
70	22,229	10,493	11,736		
80	6,502	2,942	3,560		
90	601	329	272		

* *See* previous extract, p. 465–7.

Whitechapel.					
Decennial Life Table.				Expectation of Life.	
Age.	Persons.	Males.	Females.	Males.	Females.
0	100,000	50,991	49,009	31·0	34·3
10	58,125	29,141	28,984	41·6	45·6
20	55,464	27,720	27,744	33·5	37·5
30	50,773	24,847	25,926	26·7	29·8
40	43,865	20,917	22,948	21·1	23·1
50	35,369	16,186	19,183	15·9	16·7
60	24,024	11,245	12,779	10·9	12·8
70	13,458	5,721	7,737		
80	4,004	1,345	2,659		
90	399	137	262		

when the mortality was low in both districts. The deaths in St. George's Hospital and the London Hospital were all excluded, except the proportion at the several ages due to these districts, in common with others in which there were no hospitals. (5th Annual Report, p. 443.)

Properties and Applications of Life Tables; Dr. Price's Fallacies.—The applications and uses of national life tables are almost innumerable; without an intimate knowledge of their properties it is impossible to determine the laws of population, which are the bases of statistics, or to reason upon such matters without falling into great errors, of which, if it were not invidious, too many instances might be cited from current works on population and public health. I therefore strongly recommend the student to make himself master of this subject, by a careful perusal of the writings of Halley, Deparcieux, Demoivre, Simpson, Price, Duvillard, Baily, Milne, Gompertz, Davies, Edmonds, De Morgan, Babbage, and others.

A great improvement in the life table, suggested by Graunt and invented by Halley, was made in 1806 by Duvillard. Barrett discovered the advantages of an analogous construction in calculating life annuities; Mr. Baily explained some of the uses of the new column, and showed its applicability to joint life tables in the description of Barrett's method, appended to his "Doctrine of Life Annuities and "Assurances" (1813). Mathieu has given for some years, in the "Annuaire de France," a table deduced directly from Duvillard's new column (S.*y*); it is the development, as Mathieu remarks, of a shorter table in Duvillard's work.* Mr. Griffith Davies, adopting Barrett's method, extended it, and facilitated its application to the calculation of life annuities and assurances, in a small and very useful volume —"Tables of Life Contingencies" (1825); which was to have been followed by "a more extensive work," containing "a New Theory "of the Doctrine of Annuities and Assurances,"—unfortunately never completed. Mr. Davies's views have been, however, developed in the work of Mr. David Jones on Annuities; and De Morgan has described the construction, arrangement, and use of the new tabular form in two

* Analyse et Tableaux de l'Influence de la Petite Vérole, p. 123; 1806. Duvillard states that his table, which is evidently constructed on imperfect data, was presented to the Institute, An. V of the Republic.

elegant papers inserted in the Companions to the British Almanac for 1840 and 1842.

The English Life Table shows, out of 100,000 children born alive, the respective numbers of males and females born, and the numbers attaining each age, or birth-day, from the first to the 110th, according to the rates and laws of mortality, deduced from the Returns of the Population, Births and Deaths in England (1841). Thus, in 100,000 children born alive, 51,274 are boys, and 48,726 girls; 33,060 males, and 32,464 females attain the age of 21; and 11,824 males, 12,708 females, live to 70. The males and females are not distinguished in Halley's Tables, in the Northampton Table, or in Mr. Milne's Carlisle Table. Dr. Price constructed three life tables from the Swedish observations; one for males, another for females, and a third for "males "and females collectively."* He made 10,000 males and 10,000 females the bases of his male and female tables; to which there can be no objection, except that the construction does not show the relative numbers of the two sexes born, and living together at each age. The table of "males and females collectively," is constructed from the other two tables, upon an erroneous principle, which Dr. Price lays down, and thus illustrates:—"Table 44 shows, that of 2,701 males living at "60 years of age, 560 will die in five years; and that of 3,167 females "living at the same age, 588 will die in the same time. From hence "it may be easily deduced, that of 2,930 persons living at 60, consisting "*one half of males and one half of females*, 576 will die in the same "time. The number, therefore, living at 60 will at 65 be reduced "to 2,354; which number *must again be supposed to consist one half* "*of males and the other half of females*, and the proper decrement "for the next five years deduced in the same manner from Table 44. "And it is in this method that the whole of this Table (45) has been "constructed, which, *therefore, must exhibit more accurately than any* "*other* the probabilities of living among the general mass of mankind, "consisting of males and females taken collectively." Granting that the 2,930 persons at the age of 60 consist of an *equal number* of males and females, we shall have the following results, according to Dr. Price's data:—

	Persons.	Males.	Females.
Living at the age of 60 - -	2,930	1,465	1,465
Die in the next five years -	576	304	272
Actually living at the age of 65, according to Dr. Price's facts	2,354	1,161	1,193
Living at the age of 65, according to Dr. Price's new supposition	2,354	1,177	1,177
Errors in the supposition -		+ 16	− 16

Dr. Price arbitrarily substituted 16 males for 16 females at the end of five years, and proceeded to calculate the reduction in the next quinquennial period on 1,177 instead of 1,161 males, and 1,177 instead of 1,193 females. As the mortality of males in the five years is greater than that of females (at the age 60–65, the ratio is ·207 to ·186), the mortality is exaggerated, and the same error pervades the whole table, and the subsequent tables of the value of annuities on single and joint lives in general; for it is evident that if annuities were granted to

* Price's Works, by Morgan, 7th edition; vol. ii., pp. 406–414.

equal numbers of males and females, at 60 or any other age, the excess of surviving female annuitants could not at the end of every five years be arbitrarily set aside and replaced by males.*

The Swedish Table of Mr. Milne is free from the errors† of the distinguished writer who had the merit of first constructing a national life table, and shows the numbers of males and females who complete every year of age out of 10,210 males, and 9,790 females born in that kingdom. It is, therefore, to this eminent man that we are indebted for the first correct national life table. The numbers in Mr. Milne's Table 5 multiplied by 10, and those of Table 4 multiplied by 5, $\left(\text{or } \frac{10}{2}\right)$, may be compared with the English Life Table; the basis of which is 100,000 children born alive = 51,274 boys + 48,726 girls. The numbers in the English Table are connected by simple laws, derived directly from and representing the mean results of the observed facts; it terminates, if we descend no lower than unity, naturally at 103; but, according to the same law, more than two in 10 millions born see their 108th birthday. The registers give a proportionally greater number of centenarians; but the evidence of these advanced ages is almost always unsatisfactory, and it was thought right in this instance to deviate from the observations, and to carry out the table according to the law deduced from the more numerous and more accurate statements of the earlier part of old age; the result of which is a middle course between the tables of old writers, that terminated at 84, 90, or 96 years, and the recorded instances of very advanced age. (6th Annual Report, pp. 524–6.)

Mortality in Increasing Populations.—It is frequently stated that the proportion of deaths to the population is raised in an increasing population by the excess of young children, among whom the mortality is greater than it is among adults; but this is not borne out either by experience or by theory. The mortality among males in England is 2·29 per cent.; three births are registered to two deaths, and the mortality would be 2·49 per cent. if the births and deaths were equal. The solution of the following question may throw some light upon this subject.

A population has been stationary and the mortality the same for a century. The births, which had been equal to the deaths, are suddenly increased in the proportion of 3 to 2, and remain the same, or one-half more than they had been. How many annual deaths will occur to 100 living of all ages, assuming that the law of mortality remains invariable?

Taking the English Table for males, the radix of 51,274 births will become 76,911, which will in 10 years add 192,593 ($\frac{1}{2}Q_{0|10}$) to the population, originally 2,059,501; but the deaths to the 192,593 at that age will be $= \frac{1}{2}D_{0|10} = 7{,}855$; and the ratio of the deaths to the population will be 59,129 : 2,252,094. The formula will be in all cases $\frac{D_0 + \frac{1}{2}D_{0|y}}{Q_0 + \frac{1}{2}Q_{0|y}}$; and by making y successively 0, 10, 20, 30, 40, 50, 60, 80, and 105, and taking the numbers out from the table, the following results are obtained:—

* Dr. Price's Swedish table of mortality is printed without comment by Baily and Mr. David Jones. Baily has since adverted to the error.

† Milne on Annuities, &c., Tables 4, 5, pp. 566, 569.

Years.	Population maintained by the original number of annual Births (= 51,274).	Increase of Population by the increase of 25,637 annual Births.	Total Population.	Annual Deaths in the original Population.	Annual Deaths in the new Population.	Total Annual Deaths.	Annual Mortality. Per Cent.	Annual Mortality. One in
	Q_0	$\frac{1}{2}Q_{0\|y}$	$Q_0+\frac{1}{2}Q_{0\|y}$	D_0	$\frac{1}{2}D_{0\|y}$	$D_0+\frac{1}{2}D_{0\|y}$	$\frac{100\,D_x}{Q_x}$	$\frac{Q_x}{D_x}$
0	2,059,501	—	2,059,501	51,274	—	51,274	2·490	40
10	2,059,501	192,593	2,252,094	51,274	7,855	59,129	2·626	38
20	2,059,501	365,238	2,424,739	51,274	8,975	60,249	2·485	40
30	2,059,501	524,925	2,584,426	51,274	10,400	61,674	2·386	42
40	2,059,501	669,163	2,728,664	51,274	12,065	63,339	2·321	43
50	2,059,501	795,633	2,855,134	51,274	13,949	65,223	2·284	44
60	2,059,501	901,922	2,961,423	51,274	16,233	67,507	2·280	44
80	2,059,501	1,019,083	3,078,584	51,274	23,479	74,753	2·428	41
105	2,059,501	1,029,751	3,089,252	51,274	25,637	76,911	2·490	40

The mortality of the new population in the first 10 years is much greater than ·02490; after 20 it becomes less, and the aggregate mortality remains less than ·02490 until the constitution of the population, in respect to age, is restored to its original state.

If the births, from any cause, increased in a geometrical progression, and the rate of increase were r annually, the column D would become $(1+r)^1 \cdot D_{105} + (1+r)^2 \cdot D_{104} + (1+r)^3 \cdot D_{103} + \ldots (1+r)^{100} \cdot D_0$; and all the other columns of the table being derivable from D, it is evident that, the law of mortality remaining the same, the numbers dying (C_x) and living (P_x) would, relatively to the total numbers, be increased at the earlier ages, while the proportion of deaths to the population would be diminished. If the increase were temporary, the contrary result might be produced.

The annual mortality of persons of all ages, and of persons of the age of 20 and upwards is nearly the same, for $\frac{D_0}{Q_0}$ is nearly equal $\frac{D_{20}}{Q_{20}}$. If the population of a city were for a century recruited partly by emigrants from the country at the age of 20, the proportion of deaths to the population would not be much disturbed by that circumstance; but, if the immigrants entered at 25, the apparent mortality would be increased, for $\frac{D_0 + D_{25}}{Q_0 + Q_{25}} > \frac{D_0}{Q_0}$.—(6th Annual Report, pp. 534–5.)

Statistical Methods for determining the relative Health and Mortality of different Classes of the Population.—It is universally admitted by persons acquainted with the subject, that the relative mortality of a mixed population, consisting of persons of all ages in different proportions, can only be accurately determined by ascertaining the proportion of the deaths, in a given time, to the living at the several quinquennial, or decennial periods of age; and that the mean duration of life attained by such a population can only be positively and conveniently ascertained by a construction which is called a "Table of Mortality," or a "Life "Table." The two series of facts—(1) the living at different ages, and (2) the dying at the same ages, are not always known; and two methods of approximation have been proposed and employed in such cases with various success.

The first method is this: The ages of those who die are added up, and the sum is divided by the number of deaths; the quotient is the "mean age at death." The health of two populations and their mean lifetime are supposed to be in the ratio of the "mean age at death" so obtained. This method was first employed in the 17th and 18th centuries, before any Census of the population was taken; and certain corrections of the errors in its results were proposed by Dr. Price in the construction of the Northampton Table; which, deduced from the deaths at different ages in All Souls' Parish, gave the "mean age at "death," and not the "expectation of life," in Northampton.

The other method of determining the relative mortality which has been employed in the present century by statists is equally simple: the mean population is divided by the annual deaths; or the proportion dying in a year to 100 living of all ages is found; and the relative mortality of two districts, or counties, is thus compared.

The two methods are subject to error from the disproportion in the numbers of young and old people, which may arise from the marriages being earlier or later, from emigration, immigration, and a great variety of causes, besides the mortality. In an increasing population, with an excess of children and young adults, the "mean age at death" is reduced in a certain ratio to the rate of increase, and there is nothing to neutralize the tendency or to diminish the error. By the second method, as the mortality is highest in the first year of life, and lowest about the age of 10–15, there are two elements in an increasing or decreasing population, acting in an opposite direction; the introduction of an excess of children under four years of age tending to raise the aggregate mortality, on the one hand; on the other, the excess of young persons above five years tending to depress it below the average. The inquirer who has made himself master of the nature of a life table, or even takes the fact just stated into account, can have no difficulty in deciding upon the relative merits of the two methods.

Attention has been latterly attracted to the health of towns, and of particular parts and streets; and, as it may be hoped that the vast importance of these difficult local inquiries will induce those who have entered upon them to prosecute their researches with renewed vigour, I have endeavoured to state, in a popular manner, the best method of determining the relative mortality, and to point out the fallacies to which the old method of the "mean age at death" is peculiarly liable.

If 100,000 persons born at the same moment were followed through life, the numbers that died in each year of age noted, and the sum of their ages divided by 100,000, the average ages which they lived would be obtained. It would be the mean duration of their lives, or what is often called their "expectation of life." Say that it is found to be 41 years. If another 100,000 were taken in worse circumstances, dealt with in the same manner, and their average duration of life were found to be 26 years, you infer that life is shortened 15 years in the latter circumstances.

By taking the population living in the middle of a year (1841 for instance) at each age—0–1, 1–2, 2–3, 3–4, 4–5, 5–10, &c., and the deaths in the same year at the same ages, we find how many die in each year of age out of a given number living; and can calculate, therefore, how many will arrive at the age of 1, 2, 3, 4, 5, 20, 30, &c. years; or determine the true mean duration of life. This was the method which Dr. Price pursued in framing the Swedish Table. This was the method which Mr. Milne pursued in framing the Carlisle Table. Everybody admits that this method gives as correct a result as can be obtained.

If there is no emigration or immigration, and the births and deaths are nearly equal for 100 years, the "mean age at death" will coincide with the "mean future lifetime" or the expectation of life. Thus, if the births and deaths had for a long time been equal in England, all persons born had died in it, and no strangers had entered, or if those who entered were of the same age as those who emigrated, "the mean "age at death" would be 41 years; but the births exceed the deaths more than 50 per cent., and the "mean age at death" in England, instead of 41 years, is 29 years, while it was 33 years in Mr. Rickman's time!

When tested, the error in the result by this method is 12 years in 41 years!

But it may be asked, Will not this method serve as a means of comparison, where there are not the data, nor skill, nor time requisite for calculating the true duration of life? Does not this method give results more accurate than those deducible from a comparison of the "proportion of the deaths to the population?" To this the answer must be in the negative. Neither method gives the true mean duration of life. Nobody pretends that the latter does; but it gives the nearest approximation, for, as a general rule, wherever the mortality is high the duration of life is low, and the reverse. I know no exception to this rule. In the last Report instances were given in which the indications of "the mean age at death" are altogether erroneous; thus, the "mean "age at death" is higher in France, where the true duration of life is lower and the mortality higher than in England.

	1	2	3
	Mortality: or one Death	"Mean Age at Death."	Mean Lifetime; or the "Expectation of Life."
England (1841) - -	In 46 living	29 years	41 years.
France " - -	42 "	34 "	40 " (?)
Sweden " - - -	41 "	31 "	39 "
Metropolis " - -	41* "	29 "	37 "
Liverpool " - - -	30 "	21 "	26 "
Surrey (extra-Metropolitan) -	52 "	34 "	45 "

* One in 39 was the average of five years, 1838–42.

The last column gives the true mean duration of life; where this has not been determined, the method, of which the first column contains the results, gives an approximation, sufficiently near for many purposes, to the relative mortality. Thus, according to that column, we should arrange the six classes of people in the following order of healthiness:—

(1) Surrey, 52; England, 46; France, 42; Sweden, 41; Metropolis, 39 (average of five years); Liverpool, 30.

And this is the order in which they are placed according to the expectations of life, which are as follows:—

(2) Surrey, 45; England, 41; France, 40; Sweden, 39; Metropolis, 37; Liverpool, 26.

Let us test the method of the "mean age at death" in the same way, by substituting the numbers in the second column for the others:—

(3) Surrey, 34; France, 34; Sweden, 31; England, 29; Metropolis, 29; Liverpool, 21.

Now, according to this method, the French live five years longer than the English, and the duration of life is the same in the metropolis as in all England. The method, like a rough and very bad instrument, gives you some idea of the thing which it pretends to measure, but its indications are, in many cases, entirely wrong. It neither gives the true duration of life, nor the relative duration of life in different circumstances.

Up to the date of the first edition of Dr. Price's book, the method (of which that of the "mean age at death" is a rude fragment), was the only one in use; the *deaths* could not be compared with the *population* of London in the 18th century, because the population had not been enumerated. Dr. Price's Table, and the previous Table of Simpson, gave the "mean age at death," as deduced by them from the London Bills of Mortality,* with a correction for the adults who came to settle in London. Dr. Price believed that the births did not, in his time, exceed the deaths in England, and upon this hypothesis, which he supported very ingeniously, the "expectation of life," and the "mean "age at death," would have been the same. Halley, Simpson, Price—all the great writers on the subject in the sixteenth and seventeenth centuries—could not obtain returns of the ages of the living, and had to calculate their tables from the deaths alone as given in the Bills of Mortality then in use. To avoid the errors introduced by comparing the ages at death of populations in which the births and deaths are not equal, they expressly selected towns, &c., in which they were led to believe the births and deaths were nearly equal. As the populations of England and Europe formerly increased very slowly, their tables (of Breslau, London, Northampton) are not very erroneous at advanced periods of life.

Mr. Milne's views† have been misunderstood and misrepresented; he distinguished (1) The "mean life," or "expectation of life." (2) The mortality expressed by the numbers out of which one death occurred annually; and (3) The mean life deduced from the deaths alone, or the "mean age at death." Thus in 1801–5 the true "mean duration of life" in Sweden, he says, was 39·39 years; *one* died annually in 40·90 living, and the "mean age at death" was only 30·86 years in the same period. Mr. Milne shows, from ten examples, that the two first sets of numbers are the same in a stationary population and differ slightly in an increasing or decreasing population. He shows that the "ages of death" differ from these two sets of numbers in an increasing population; thus in Sweden the difference was 40·90 − 30·86 = 10·04 years. In other instances the difference amounts to 6, 7, 9, 10 years; in England it is 12 years.

Mr. Milne says, "When tables of mortality are constructed from the "*numbers of deaths only* in the different intervals of age, without comparing them with the *numbers of living persons* in the same intervals " * * * * and the population is increasing, the number of years in "the mean duration of life from birth ('ages of death') will fall short "of the number of the people out of which one dies annually ('propor-"'tions of deaths to the population') by *a much greater number* than in "the case we have just been considering, of the table of mortality having "been properly constructed from the necessary data, as the following "statement will show." (Here follows a table with 10 examples showing the differences.)

* Price's Works, by Morgan—the London Tables; also Simpson on Annuities, p. 1.

† *See* Art. "Mortality," in Ency. Britannica.

Mr. Milne's Carlisle Table of mortality,* as he explains, represents a *population* in which the births are equal to the deaths, and in which there is neither immigration nor emigration, such as everybody knows does not exist in nature. All correct life tables are constructed upon the same hypothesis, and *therefore admit of comparison*. Tables might be constructed representing populations in which the births exceeded the deaths, or the contrary; but they would not admit of comparison unless the births bore the same proportion to the deaths in the two cases. Such tables would not show the survivors at each year of age out of a given number born, or a given number attaining any age; the probable duration of life could not be calculated from them, nor the mean duration of life.

The "adjustment," which makes a life table represent as nearly as possible the progress of a human generation year by year through life, has been employed upon the same principle that astronomers "*reduce* "as it is termed, all their observations, both of right ascension and "declination, to some common and convenient epoch." * * "By the "term correcting or *equating* the observation for nutation," says Herschell,† "is always understood, in astronomy, the getting rid of a "periodical cause of fluctuation, and presenting a result, not as it *was* "*observed*, but as *it would have been observed*, had that cause of "fluctuation had no existence."

The method of the "mean age at death," taking the crude results of observation on a population subject to fluctuations in births and deaths, immigration and emigration, only furnishes true results in a perfectly stationary population; while with the tables deduced from the proportion of deaths at each age to the living at each age—the method employed in the construction of the Swedish, Carlisle, and English Tables—"Whether the population be stationary, or increasing, or decreasing, "and whether such changes be produced by procreation, mortality, or "migration, or by the joint operation of any two or more of those "causes, provided that the mode of their operation be uniform, or "nearly so, and not by sudden starts, the law of mortality may be "approached near enough for any useful purpose by actual *enumeration* "and the *Bills of Mortality*."‡

Several illustrations were given in the last Report of the errors produced by applying the "mean-age-at-death" method to fluctuating populations. In many cases, the mortality and unhealthiness of two classes of persons are inversely as the mean age of death. Without entering into any mathematical details, I will now show that the "mean "age of death," considered as a measure of health, must give erroneous contradictory, exaggerated results.

The mean age of death is determined by adding up the ages of all the persons whose deaths are registered in a parish or other register—say, during a year—and dividing the sum of the ages thus obtained by the number of persons who lived those ages. The error consists in the assumption that this average age represents the average age which the inhabitants (the same rate of mortality prevailing) will live, or the average age which would be obtained by following 1,000 children, born in the parish, through life, adding up the ages which the 1,000 attained, and dividing the sum by 1,000. The results of these two methods are the same in a stationary population, and totally different in an increasing, decreasing, or migratory population. The population is increasing in

* Art. "Mortality," Ency. Britannica.

† Herschell, Astronomy, p. 174.

‡ Milne.

almost every district of England, and there is a constant flow of population to the towns. To show the effects of these movements, I give two or three extreme instances.

(1.) Let us suppose that in two contiguous parishes, equally healthy—A and B—all the children are born in A and remain there up to the age of 20, and if they die are registered in the parish register; that at 20 all the survivors emigrate year by year to B, and when they die are registered by the clergyman of B in his register. Now if the mean age of death were taken in the parish A, it would be about *four* years (according to the present rate of mortality in England), while it would be 60 years in the parish B. The numbers living, out of which one death would take place annually, would be 1 in 43 in A and 1 in 40 in B. The mortality and healthiness, taking the difference of age into account, would be the same in A and in B. Notwithstanding the difference of the mean age at death produced by emigration, a comparison of the deaths with the living at each age would demonstrate, that although the *deaths of none above twenty* were registered in A, *two* in every *three* born in A survived the age of 20, and died at more advanced ages elsewhere. In B, as none died under 20, it would be inferred that none entered the parish under that age.

The emigration of a part of the adult population to towns produces an effect of precisely the same kind, though to a less extent.

(2.) In the former illustration, the births were supposed to be equal on an average to the deaths in A and B; and the error in the mean age at death would be got rid of by combining the deaths in A and B in one table, instead of placing them in contrast. Suppose the two parishes united—that after a stationary stage, through the women marrying earlier and in greater numbers, the births rise from 1,000 to 2,000 annually—while the salubrity, and the mortality at the respective ages remain precisely the same: what would be the effect of an abstract of the parish registers 20 years after the increase had set in? Why that, without any increase in the mortality, the deaths under 20 would be doubled; and the mean age at death, instead of being 41 years, would be less by many years. The comparison of the past and present mean ages at death in the united parishes would be absurd; so would the comparison with any other parish in which the increase and emigration had proceeded differently. But upon comparing the deaths with the living at each age—as by the hypothesis the living under the age of 20, as well as the deaths under the age of 20, would be found twice as numerous in the year as they were 20 years ago—the ratio between the two would be found to be the same; the survivors year by year, and their expectation of life, could be determined. With regard to the proportion of deaths at all ages to the living at all ages, a sort of compensation would be produced—the excess of infants under 5 years of age tending to raise the relative number of deaths; the excess of persons from 5 to 50, and the diminished proportion of persons above 60, having a tendency exactly the reverse.

In England, during the whole of the present century, there have been more than 3 births to 2 deaths; and the result, as has been shown, has reduced the mean age of death to 33 years in 1831, and to 29 years in 1841; while the real duration of life—the expectation of life—is 41 years, and has varied little. The number living to 1 death was 46 in 1841, and 45 in the preceding four years.

(3.) Take a street (C) in a town, where, from the erection of new factories, or from any new field of labour being thrown open, a considerable number of young men and women have been attracted within the last 10 or 15 years; there is a demand for the labour of children; marriages take place; nearly all the young couples have children, two, three, or four in a family. Take another street (D) inhabited by artizans, whose business and numbers have remained nearly stationary, and tradespeople who have succeeded to old shops established by their fathers;—Suppose the salubrity of the two streets, and the rate of mortality at the corresponding ages, the same,—it is evident that as the street C contains no old people, and the mortality in the first two or three years is always relatively high, the deaths registered will be at early ages—the mean age of death low; while in the street D, the deaths will many of them be at old ages, and the mean age at death relatively high. If all the inhabitants of the two streets died in one year, the mortality would be the same. Yet the mean age at death would differ in the same ratio as the mean age of the living. The same results would be produced by the death of *one thirtieth* of the inhabitants in each street. The cases which have been put will enable us to understand such a case as is said to have occurred in Leicester, where the mean age at death was $13\frac{1}{2}$ years in the undrained streets, and $23\frac{1}{2}$ in the drained streets. That the real mortality was higher in the one class of streets than in the other is probable; but this is not proved by the method, for the *undrained streets* may be new streets, inhabited by young people—a part of the 8,600 in 46,000 not born in Leicestershire; while the drained streets may be old streets inhabited by the old inhabitants of the town.* On account of the system of compensation which it involves, the method of comparing the total deaths to the population of the streets gives results nearer the truth; but no one acquainted with inquiries of the kind would place much confidence in any other method, as applied to *particular streets* or *small districts*, than that upon which the Life Table is founded—the comparison of the numbers living with the numbers born and dying at the several periods of life. In the Registrar-General's Report, the mortality is only given for statistical districts of an average population of 50,000.

The correct method, to which I have so often adverted, yields uniform consistent results; and next to this in accuracy is that obtained by the proportion of the deaths to the living among numbers, or over a time, sufficient to obviate errors liable to be introduced by accidental fluctuations.

The papers of Mr. Edmonds in *The Lancet*† may be referred to as early models of the methods of determining the relative mortality of particular localities, and at particular ages, both with complete and imperfect data. *See* also Mr. Chadwick's Paper "On the best mode of "representing accurately, by statistical returns, the duration of life," in the *Statistical Journal*, vol. vii., p. 1; and the following Paper in the same Journal, p. 40, "On a method recently proposed for conducting "inquiries into the sanitary condition of various districts," by Mr. Neison, who has given several striking illustrations of the results of the new method.—(6th Annual Report, pp. 570–6.)

Relative Duration of Life among Males in Manchester and in England.—The mortality of Manchester and Liverpool is nearly the same, or only differs at some ages; it is rather the highest at certain ages in the former town. The enormous extent of the mortality in both these large places is appalling. The excess over the mortality of Surrey shows to what an extent it is unnatural and susceptible of remedy.

* I find, upon turning to the Census Returns, that the populations of some of the new and old streets in Leicester differ in the manner described.

† See *The Lancet* volumes for the years 1833–39.

RELATIVE DURATION of LIFE among MALES in MANCHESTER and in ALL ENGLAND.

Precise Age.	Manchester.	England.	Manchester below the Average.
	Expectation of Life.		
	Years.	Years.	Years.
0	24·2	40·2	16·0
1	33·1	46·7	13·6
10	40·6	47·1	6·5
20	33·3	39·9	6·6
30	26·6	33·1	6·5
40	20·6	26·6	6·0
50	15·2	20·0	4·8
60	10·3	13·6	3·3
70	6·8	8·5	1·7
80	4·6	4·9	*
90	3·2	2·7	
100	1·2	1·5	

* The facts for MANCHESTER are too few to admit of a comparison after the age of 80.
The mean duration of life among Males in MANCHESTER is 24·2 years, or 16·0 years less than 40·2 years, the duration of life in all ENGLAND.
By another method the expectation of life at birth is 25·5 years in MANCHESTER.

Many of the irregularities in the columns of the life table from which the expectation of life in Manchester given in the above table has been calculated are from erroneous statements of age; the ages guessed at being almost always referred to the round numbers 20, 30, 40, 50, &c., make the numbers in the corresponding quinquennia excessive. These errors are not of great importance, and have not been corrected in this table, which, by a very simple arrangement of the facts, enables us to answer very readily many questions relative to the population of Manchester. The mean age, for instance, of the people of Manchester, males and females, is 25 years; as is shown by dividing the number of years lived (4,141,701) by the number of persons (163,561). The mean age of persons above 20 is 38 years. Other properties and uses of the Table are sufficiently obvious.—(7th Annual Report, pp. 329–39.)

Expectation of Life; *afterlifetime*.—*Expectation of life* is an incorrect term: the time which it is *expected* a person will live is the time which it is an even chance he will live; it is the *vie probable* of the French, and is correctly expressed by "probable lifetime." The afterlifetime can only be the same as the probable lifetime on Demoivre's hypothesis—that the surviving form an arithmetical progression. The term "expectation of life," first used by Demoivre, is correct, on that supposition, which is, however, in itself quite erroneous. The idea intended to be expressed by "expectation of life" is the *mean time* which a number of persons at any instant of age will live after that instant: it is the French *vie moyenne;* and this technical idea is strictly and shortly expressed by *afterlifetime*, a pure English word, formed on the same analogy as *afterlife, aftertimes, afterage, afterhours.* See the words in Johnson. Among the examples he quotes are,—

"What an opinion will *afterages* entertain of their religion?"—
Addison.

"So smile the heavens upon this holy act,
That *afterhours* with sorrow chide us not."—*Shakespeare.*

"You promised once a progeny divine
Of Romans, rising from the Trojan line,
In *aftertimes* should hold the world in awe,
And to the land and ocean give the law."—*Dryden.*

Todd adds "afterlife" from Dryden, Heywood, and Butler. "Afterlife: the remainder of life."

"All of a tenor was their *afterlife*,
No day discolored with domestic strife."—*Dryden.*

The *afterlifetime* of men at the age of 30 is 33 years by the English Life Table: 33 years is not the precise time probably that any one of that age will live, but the average time that a number of men of that age will live, taken one with another. *Age* + *afterlifetime* = *lifetime.* At 30 this is 30 + 33 = 63, the average age which men now aged 30 will attain. At birth this is 0 + 40 = 40; when *lifetime* and *afterlifetime* are the same thing. The *lifetime* simply, without the addition *at a given age*, will serve to express in one word what is often improperly called *the expectation of life at birth:* thus the *lifetime* of males in England is 40 years, the *lifetime* of males in Manchester is 24 years. Those who from habit prefer "expectation of life," can always substitute it for *afterlifetime;* from the use of which in this paper no ambiguity can arise.—(8th Annual Report, pp. 279–80.)

English Life Tables, Nos. 1 *and* 2.—The Ninth Report contained an elaborate series of tables, showing from the returns of deaths in the *seven years* 1838–44, and from the Census returns of 1841, the *mortality* of *males* and *females* at *different ages* in England and Wales, as well as severally in the 11 divisions, 44 counties, &c., and 324 groups of districts. The population was 15,914,148; the deaths in the seven years 2,436,648. A second English Life Table for males during this period has been constructed, as well as several new series of tables of use in all the ordinary operations of life insurance. The Life Tables (1 and 2), it will be seen, agree very closely, although the one is constructed on the deaths in 1841, the other on the deaths in the seven years 1838–44; the population of 1841 serving as the basis of both.

RESULTS deduced from the TWO ENGLISH LIFE TABLES (Males).

—	Ages. 0	10	20	30	40	50	60	70
Afterlifetime or Expectation of Life:—								
(1841) English Table, No. 1	40·17	47·08	39·88	33·13	26·57	20·03	13·59	8·52
(1838–44) „ No. 2	40·36	47·47	39·99	33·21	26·46	19·87	13·60	8·55
Annual Premium to insure 100*l.*:—	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.
English Table, No. 1 -	2 5 10	1 5 0	1 11 11	2 1 1	2 14 7	3 17 6	6 2 7	10 3 2
„ No. 2 -	2 5 7	1 4 5	1 11 8	2 0 9	2 14 11	3 18 6	6 2 7	10 2 0
Present Value of Annuity of £1 (payable at the end of every Year):—								
English Table, No. 1 -	18·2167	23·0333	21·1765	19·1347	16·7209	13·7365	10·0598	6·6516
„ No. 2 -	18·2660	23·2042	21·2331	19·1943	16·6689	13·6272	10·0572	6·6837

Various methods of graduation were tried. The abstracts of the population for 1841 distinguish the numbers living at each quinquennial period of life, and the abstracts of deaths are taken at corresponding periods, except in the first five years of age, when the rate of mortality varies so rapidly that every year and even month is marked by a change. But by a careful examination of the facts it was found that the rate of mortality in decennial periods, after the age of 15, furnishes the most satisfactory basis for determining the series of fractions to express the probabilities of life. At the earlier ages the mortality for each year was directly deduced from the returns, and, after careful comparison, the first series of logarithms was drawn gradually into the second—the second into the third. At another time I hope to be able to discuss in detail the various methods of graduation, and to describe that which was found most successful in practice. (12th Annual Report, pp. i–ii.)

Old and New Northampton Life Tables; Carlisle Table; Experience Table.—Life tables have been constructed in two ways:—

(1) By a comparison of the *deaths* and the *living* at each age, which gives the rates of mortality and survivorship. Tables so constructed, in the words of Dr. Price, "must be correct."

(2) From the deaths alone, or with reference only to the ages at which the deaths have taken place. Tables so constructed are only correct if the "population" of the place among whom the deaths occur is stationary, if the births and deaths are equal, and if there is no disturbing migration for a century. It was in this second way that the Northampton Table, from the want of better materials, was constructed by Dr. Price.

I was enabled to obtain data for constructing two new Northampton Tables; one on the plan of the Northampton Tables by Dr. Price, in common use, the other on the plan which "must be correct."

It is a remarkable circumstance that the *new* Northampton Table (2) which was constructed, like that of Dr. Price, on the deaths alone, yielded similar results. The mean lifetime, or duration of life, by the table of Dr. Price was 25·18 years, by the other table 24·88 years; and the same agreement in the resulting expectation of life, the values of annuities, and the premiums of insurance, is maintained at all ages.

But the correct Northampton Table, deduced from the Census of the living at each age in 1841, and the deaths in the seven years 1838–44, differs entirely and largely from both the other tables; thus, the mean duration of life in Northampton is found to be 37·57 years, or thirteen years longer than it is given by the untrue table; and at all the earlier ages the values of annuities and the rates of premiums, by the true table, agree generally with the English and other tables, constructed from adequate data, but differ totally from those based on the old Northampton Table, on which so large a part of the insurance business of this country has hitherto been transacted.

It is shown in the paper referred to that the lifetime in Northampton was about 30 years when Dr. Price's observations were taken, whereas it is now 37½ years*; that the town contained then, as it does now, great numbers of Baptists, who repudiate infant baptism, and thus, consequently, by reducing the ratio of the christenings to the births, induced Dr. Price to believe that the population was stationary, although, as shown from other sources, it was, like the staple shoe trade of the place, constantly increasing. Dr. Price assumed that the population of

* Reg. Gen. 8th Report, pp. 277–348.

the parish was kept up by immigration, and that all the immigrants entered at the age of 20; as a correction for this disturbance he was induced to alter his facts, and the alteration had the effect of increasing the error of the original table.

Dr. Price had not the data for constructing a true Northampton Table; for this reason he failed. He constructed from proper data a Swedish Table, which is nearly correct; and he recommended, in the first instance, his Chester Table, which is less erroneous than the Northampton Table; but the directors of the Equitable "judged it less safe."

The deaths, Mr. Morgan says, were afterwards found to be one third *less* in the Equitable experience than the Table indicated.

Great injustice has been done by the use of this Northampton Table; which, in mutual offices makes one member pay 40, 30, 25, 20, or 10 per cent. more than the premium which is required to secure a policy of the same value, and distributes the surplus thus acquired unequally. The old offices, which have used the Northampton Table, have a great difficulty in setting themselves right. By its use the proprietary offices have exacted enormous and unequal premiums from the portions of the community who happened to be ill-versed and ill-instructed in the intricate science of life insurance.

A false life table can be defended by the same arguments as a depreciated currency; and the substitution of a correct table causes the same kind of disturbance in the value of the shares of members as a re-coinage of clipped money, or a return from a depreciated paper to a metallic currency, introduces into the value of commodities and securities. The Northampton Table has still silent adherents, but few open defenders; and some of the old offices have, greatly to their credit, since the error in that table has been placed beyond doubt, abandoned its use.

The Carlisle Table was deduced from two enumerations of the population of the parishes of Saint Mary and Saint Cuthbert, Carlisle; the first in January 1780, when the inhabitants were 7,677 and the second in December 1787, when the inhabitants amounted to 8,677, namely, 3,864 males and 4,813 females. The deaths in the two parishes were 1840, males 881, females 959, in the *nine* years 1779–1787. It is now well established that the mortality in towns is understated at the age 15–35, when they are entered by healthy immigrants from the country; of whom many reside while they continue healthy in comfortable "situations" as domestic servants, and when they are attacked by consumption return to the country to die. The fact that the females in these parishes exceeded the males by nearly 1,000, and the great excess in the number of persons (1,501) of the age of 20–30 over those in the town of the age of 30–40 (991), indicate the character of the population with tolerable distinctness. Some of the irregularities in the graduation of the Carlisle Table may be referred to the limited extent of the observations; for the deaths in each decennial period (20–90) only ranged from 89 to 173 in number.

The "*Experience*" *Table* is the result in part of a highly praiseworthy effort which was made by a Committee of Actuaries to collect all the extant observations on the mortality of persons whose lives have been insured. The *Equitable* and the *Amicable* Life Offices had before published their experience; and had thus offered a valuable and liberal contribution to the science of life insurance. The Committee of Actuaries induced the following 15 offices out of more than a hundred then existing, to contribute but not to publish their experience:—the Alliance, British, Commercial, Crown, Economic, Guardian, Imperial, Law Life, London Life, Norwich Union, Promoter, Scottish Widows Fund, Sun, Universal, and the University.* (12th Annual Report, pp. iv–vi.)

Selection of Lives for Insurance.—When life insurance was first commenced, in the absence of experience it was uncertain whether the rate of mortality among persons who insured their lives would be higher or lower than the rate of mortality prevailing in the nation generally of which they formed a part.

Dr. Price anticipated an excessive rate of mortality in insurance societies, for he says:—"Those persons will be most for flying to these "establishments who have feeble constitutions, or are subject to "distempers which they know render their lives particularly precarious; "and it is to be feared that no caution will be sufficient to prevent all "danger from hence."†

Mr. W. Morgan in 1829, after fifty years experience as actuary in the Equitable Life Office, thus expresses himself on this subject:—

"Between a number of select lives and the general mass of mankind "the difference in the rate of mortality will at first be considerably "in favour of the former; but this difference will be continually "lessening, till in process of time it will vanish altogether, as it is "found to have done among the lives of long standing in the *Equitable* "*Society*. Should any institution, therefore, of this kind, in the early "period of its existence be tempted, by the higher probability of life "among its members, either to reduce its premiums or to adopt any "other violent measures for impairing its resources, the consequences, "though not immediate, must ultimately terminate in disappointment "and ruin."‡

Mr. Milne, after much experience, also observes on the same subject: § "Although the members of such a society [as the Equitable] when "they first enter are select lives, they are not even then so much "better than the common average as many persons suppose; for the "more precarious a life is, the stronger is the inducement for parties "interested in its continuance to get it insured, so that bad risks are "frequently offered to such companies. And many proposals for insurance "are accepted by the directors that are not thought very eligible at "the time, in cases where they are not aware of any specific objection "to the life proposed. Besides, it is to be considered that of the "number in a society at any one time but a small proportion can have "been recently admitted, and in a few years from the time of admis- "sion *the members* will generally have come down to the *common* "*average of persons of the same ages*."

* A few copies only of the "Experience" Table were printed and circulated, it appears exclusively among the insurances offices. Unlike the Equitable and the Amicable societies, the Directors of the insurance companies have not yet evinced such an anxiety to promote the science of life insurance, or to supply the English public with information, as might reasonably be expected from a class so enlightened and so anxious to promote the general good. The few facts in the text are derived from the subjoined works, and they agree closely with the similar results derived from the Equitable experience by Mr. Morgan, and recently by Mr. Peter Hardy:
Life Contingency Tables. By Edwin James Farren. Part I.
A Series of Tables of Annuities and Assurances. By Jenkin Jones.
Insurance Magazine.

† Works, vol. i. pp. 176–7.

‡ A View of the Rise and Progress of the Equitable Society. By W. Morgan, F.R.S., 1829, p. 46.

§ Human Mortality, Encyclopædia Britannica, vol. xv., part II., page 555.

The effect of selection is now well known, and the facts have been investigated by Mr. E. Farren, who has separately deduced the mortality among insured lives during the first year, and the mortality among the same lives in the first as well as subsequent years; he has also compared the results with those derived from the first English Life Table.

The mortality per 1,000 at 5 ages is thus given.*

Age.	Among insured Lives.		Among the general Male Population of England.
	First Year.	First and subsequent Years.	
30 - - -	6·584	8·732	10·220
40 - - -	8·588	10·796	13·195
50 - - -	14·595	16·398	16·994
60 - - -	28·368	31·082	31·441
70 - - -	54·382	62·676	67·001
	A	C	P

After eliminating the influence of selection over the first year Mr. Farren concludes that the rates of mortality of persons insured "would not particularly differ from those prevailing among the male "population at large, taken indiscriminately without regard to health."

The observations of the most distinguished writers, these recent investigations, and the nature of the case, clearly show that in the present state of our knowledge the National Life Table—or a table derived from the same facts—is the soundest and justest basis of the life insurance business of the country; for if there is a disposition to pass an undue proportion of "bad lives" into an insurance society, such arrangements can be made as, when skilfully conducted, reduce the mortality below the average of mankind in general. Any favourable result of this application of technical skill may be set down as legitimate *profit*, for if such skill is not applied the mortality will be above the average of the nation, and the result "loss."

The selection of lives is not yet fully understood, but it may be broadly stated that 27 in 1,000 men of the population, of the age of 20 and under 60, are suffering from some kind of disease or other; that several of the diseases are of long duration, that others are recurrent, and that some are hereditary; that consumption, the most common fatal disease, lasts on an average two years, although it varies considerably in duration; and that cancer, another form of chronic fatal disease, is much more common in women than it is in men.† On all these grounds it is evident that selection will diminish the mortality in the first year, or two or three or four years subsequent to its exercise. As age advances the influence of selection increases; and in this way it is stated that a former Government incurred heavy losses by the sale of life annuities on old lives to unscrupulous speculators.

Investigations seem to indicate that selection under the existing tests is more effective in the case of females than in the case of males; and that it is not safe at present to insure females at lower premiums than males. (12th Annual Report, pp. vi–xi.)

* The Chances of Premature Death, and the Value of Selection among assured Lives. By E. J. Farren, 1850, Part I., p. x. and p. xiii.

† Walshe on Cancer.

Rise and Progress of Life Insurance.—Up to the 17th century the population of England experienced at intervals periodical plagues which destroyed a third or a fourth part of the population of London and of other cities; at the same time the interest of money continued 6 per cent. per annum, while it was 3 per cent. in Holland and Italy—a decisive proof, in addition to historical examples of the bad faith of the times, that in this case, as in nearly all others, the excess in the rate of interest over 3 per cent. is no more than covers the risk of the *class of securities* on which the high rate of interest is charged. The average duration of human life, and its modifications by age, were then unknown. In such a state of things life insurance on any solid rational principles was impossible; nor is there any proof that the social condition of the people of the 17th or any previous century, was such as would induce them to insure their lives extensively for the benefit of their families. After the revolution, and the establishment of the Bank of England, the financial state of the country became settled; and the Government borrowed money on life annuities but on very disadvantageous terms. Men began to have confidence in each other, and the way was opened to investments for long terms. Halley constructed a life table, and inserted it in the Transactions of the early and glorious days of the Royal Society. Demoivre, Simpson, and Dodson, in their works, showed its practical application to all questions in which money payments are dependent upon human life; and the doctrines of life insurance were extended by the financiers of Holland, Germany, and France. Sweden contributed its national tables. Dr. Price, Mr. Morgan, and the directors of the Equitable Society, in various ways, gave a great impulse to life insurance at the close of the 18th century. Mr. Pitt mitigated the injustice of his income tax by exempting from taxation that part of the income which was paid as premium for insurance; a modification in favour, he said, "of those who have recourse to that *easy, certain, and* " *advantageous mode* of providing for their families by insuring their " lives."* The fire offices commenced insuring lives. The Equitable advanced rapidly, and new offices were established. Baily enumerated 15 life insurance companies in 1813; Mr. Babbage analysed 32 in 1825, thirty of which engaged the attention of 528 directors. The science received considerable accessions by the writings of W. Morgan and Baily. In 1815 Mr. Milne's admirable work appeared, containing a systematic and original digest of the science of life contingencies, which has been extended, modified, and enriched by the analyses and contributions of Barrett, Griffith Davies, De Morgan, Gompertz, Edmonds, Galloway, A. Morgan, Ansell, Neison, Jones, Sang, and other English as well as foreign writers. The number of offices which now exist exceeds 150; and the knowledge of insurance is extending in every direction.

The whole of the commerce of the country turns on contingencies which demand the application of scientific observation and calculation; and as English agriculture has its chemists, English commerce must—to keep pace with it—ultimately employ actuaries, to calculate the risks, which are now only roughly guessed at; and thus extend the useful sphere of an important scientific class of men at present almost peculiar to this country.

Her Majesty's Government, by facilitating that part of their undertaking which life offices have hitherto found to be attended with most difficulty, namely, the exhibiting of such accurate, clear, and frequent statements of their financial condition and progress as may

* Speech in the House of Commons on 14th December 1798.

inspire the public with entire confidence in the success of their arduous engagements—will contribute to their prosperity, and will tend to make England, what it has already partly become, a great emporium for the highest class of securities—policies on lives.

The families of the classes living on wages are still in nearly the same condition, as respects life insurance and a provision against infirmity and age, as the professional and middle classes in the last century; but the stimulus which the facilities and security that the Government could afford, would undoubtedly induce them, in the course of time, to make such a provision for the future as would be satisfactory to their own minds through life, preserve them from humiliation in old age, and in the event of untimely death secure as an inheritance for their widows and fatherless children a decent livelihood, instead of a life of anguish, infamy, or crime. (12th Annual Report, pp. l–lii.)

Construction of the English Life Table No. 3.—The English Life Table No. 3 was calculated in the General Register Office with the help of the Scheutz calculating machine, and was based upon the Census enumerations of 1841 and 1851, and upon the 6,470,720 deaths registered in the 17 years 1838–54. It consists of three parts, or three Life Tables, each of seven columns; the first part for Persons, consisting of such proportions at each age of the two sexes as are produced by the births; the second part for Males; and the third part for Females. The base of the Table for Persons is 1,000,000 children born alive; and as boys and girls were born in England during the period of observation in the proportions of 511,745 boys to 488,255 girls, these numbers were made respectively the bases of the Male Life Table and of the Female Life Table.

In the Synoptical Table the numbers of the males and females living and dying at each year of age are given as they would exist in a population under the law of birth and mortality, found by direct observation to prevail in England and Wales, undisturbed by emigration, by excess of births over deaths, or by any other element of that kind.

The males, we find, if there is no emigration, exceed the females in number in infancy, in childhood, and in manhood up to the age of 53, when the women after the age of childbearing enjoy a firmer hold on life, and die at a lower rate than the men; so that the number of women of 53 and upwards exceeds the number of men of the corresponding ages. The males are to the females of all ages as 20,426,138 to 20,432,046; thus proving decisively that the disparity in the numbers of the two sexes of the English population is due exclusively to emigration.

The Male and Female Life Tables were constructed independently; that of the Persons was obtained by combining the other two in one.

The Life Table is based upon the observed *rates of mortality* at different ages in England and Wales.

The rate of mortality—or, in a technical sense, the mortality—expresses the ratio between three elements: (1) men living; (2) time; and (3) men dying.

The men living, and the *time expressed in years*, multiplied into each other, produce the years of life with which the deaths are compared. A *year of life* is the lifetime unit. It is represented by one person living through a year. Any number of persons living, one at a time, *in continuous succession* through a year, yield also one year of life. There are 525,949 minutes in a year; and 525,949 persons living through one minute also enjoy *one year* of life.

The years of life are determined by enumerating the population at certain points of time, and thence deducing the numbers living through the time.

The following four cases may be distinguished :—

1. The population is known or is assumed to be *stationary*. Thus a population of *ten thousand* living through *two years*, yields *twenty thousand years of life*. Let the population be P, and the years of life y; then in x *years*, $y = x\text{P}$ in all these cases. Here x may be an integer or a fraction.

2. The population increases by equal numbers in equal times; that is, it increases in *arithmetical progression*. If the population is 4,000 at the beginning and 6,000 at the end of a period, the mean population will be the sum of these numbers divided by 2, that is, 5,000; which will also be the population in the middle of the period. 5,000 multiplied by the intervening years gives the years of life. Thus if P_0 is the population at the beginning, and P_x the population at the end of x years; $r = \frac{\text{P}_x - \text{P}_0}{x} =$ *annual* increase; for the increase (r) is equal in equal times by hypothesis. The population at the end of any time is $\text{P}_x = \text{P}_0 + rx$. And $dy = \text{P}_0\,dx + rxdx$ is the differential of the lifetime, from which we have by integration $y_x = x\,\text{P}_0 + r\frac{x^2}{2} =$ the years of life $=$ the population in the middle of the period $\left(\text{P}_0 + r\frac{x}{2}\right)$ multiplied by the years (x) over which the observation extends.

3. If the population fluctuates much, and frequent enumerations are made, the mean of each successive couple of enumerations, multiplied by the time, expressed in years and fractions of a year, will give nearly the years of life. When the numbers at the beginning and at n equal intervals of time are ascertained, the half of the extremes added to the intermediate terms makes a sum which, divided by one less than the number ($n + 1$) of enumerations, gives the mean population.

4. The population of a country naturally increases in equal proportions in equal times, or in geometrical progression; for this is a necessary result if the increase also increases. Thus, let a population represented by 1 become $1 + i$ in the first year; then :—

$$1 : 1 + i :: 1 + i : (1 + i) \times (1 + i) = 1 + 2\,i + i^2.$$

Thus the increment of $(1 + i)$ is $i + i^2$; the i growing out of the population represented by 1, and i^2 growing out of the population represented by i. The population in two successive equal intervals of time will be in the ratio $1 : 1 + i : 1 + 2\,i + i^2$; and the arithmetical mean of the first term and the third term $= 1 + i + \frac{i^2}{2}$; which in a short time, or when i is a small fraction, differs inappreciably from the middle term $(1 + i)$, for the $\frac{i^2}{2}$ may be neglected. And so it is if the mean of several terms is taken. The result, therefore, in such cases, differs little from the result which is obtained under the second head.

The division of the average annual deaths by the arithmetical mean population of the extremes, understates the mortality $\left(\frac{d}{\text{P}} = m\right)$, which varies inversely as the population. For the same reason the division by the population living in the middle of the period overstates the mortality. The shorter the period the less is the error; indeed the two values approximate indefinitely as the period is shortened, and as

the ratio (r) recedes to unity. Either divisor may be employed where great precision is unattainable, or is not required.

The mean population existing at the two points equidistant from the two extremes—in 1841 and in 1851—is near the true mean population living through the 17 years 1838–54. By taking these years the error in the years of life is reduced to a minimum.

Thus the mean population of the age 15–25 was by the different methods :—

MALES :—

$\frac{y_{17}}{17} = 1{,}591{,}550 =$ true mean population, on the hypothesis that the population increased in geometrical progression at a uniform rate.

$\frac{\text{P}_{41} + \text{P}_{51}}{2} = 1{,}591{,}618 =$ arithmetical mean of the population living in 1841 and 1851.

$\frac{\text{P}_0 + \text{P}_{17}}{2} = 1{,}595{,}424 =$ arithmetical mean of the population living at the beginning and end of the 17 years.

$r^{8\frac{1}{2}}\,\text{P}_0 = 1{,}589{,}606 =$ population living in the middle of the period.

The annual rate of mortality is determined for the several periods of life by dividing the deaths at each age by the contemporaneous years of life out of which they occur. If P represents the years of life enjoyed by men of the age of 20 and under 30, and d the corresponding deaths, then $\frac{d}{\text{P}} = m =$ the annual rate of mortality among the men of that age. $100\,m =$ the annual rate *per cent.*

AVERAGE ANNUAL RATE of MORTALITY in ENGLAND and WALES in the 17 years 1838–54.

AGES.	PERSONS.	MALES.	FEMALES.
All Ages -	·02245	·02328	·02165
0–	·06738	·07250	·06228
5–	·00916	·00920	·00911
10–	·00527	·00517	·00538
15–	·00838	·00822	·00853
25–	·01028	·00999	·01055
35–	·01277	·01283	·01270
45–	·01715	·01851	·01587
55–	·02992	·03183	·02816
65–	·06319	·06689	·05999
75–	·14027	·14758	·13437
85–	·28820	·30136	·27915
95 and upwards	·43501	·44031	·43223

The rate of mortality at each age is thus deduced from the deaths registered at that age, and from the population of the corresponding age enumerated at the Censuses. Now it is generally admitted that the ages of a certain number of women are understated; and I had to consider what correction was necessary upon this ground.

The probable extent of the error in the statements of women's ages, it was shown in the Census Report for 1851, is not considerable; but as the effect of the error is not always understood, some explanation is necessary.

The English Life Table is not deduced from the population or from the deaths alone, but from the ratio the one bears to the other at different ages; and to display the effect of transfers from one age to the other, assume that the following numbers represent the exact numbers living and dying at the three ages 25, 35, and 45.

Age.	FEMALES.		Annual Rate of Mortality.
	Living.	Dying in a Year.	
(x)	P_x.	d_x.	m_x.
25	313,095	3,024	·00966
35	281,506	3,279	·01165
45	247,434	3,555	·01437

The mortality, it will be observed, increases as age advances; so that if all the women of 35 were returned as ten years younger than they are, the mortality at the age 25 would be overstated; the excess being ·00199. But there is less chance of women of 35 being returned at death as 25 than there is of their being so returned at the Census; and the corrective effect of this excess in the proportion of women at the Census transferred to the earlier ages is apparent, on inspection of the formulas below. d represents the deaths, P the population, m the mortality, and r^{10} the increase of the rate of mortality between the ages 25 and 35. Thus—

$$\frac{d_{25}}{P_{25}} = m_{25}\,; \ \frac{d_{35}}{P_{35}} = r^{10} m_{25} \ \therefore \ \frac{d_{35}}{r^{10} P_{35}} = m_{25}.$$

It is evident that an increase in the population and a decrease in the relative deaths transferred from 35 to 25 might reduce the error to insignificance. From the numbers living at several periods of age a series representing the numbers living at each year of age was obtained by the method of finite differences; and from these numbers again the living at the ages 25–35, 35–45. . . . . were obtained. The residuary errors affecting the values m_x are, it is believed, inconsiderable.

Upon comparing the female rates of mortality with those of males, and the rates of progression in the mortality of the two sexes, I have come to the conclusion, after carefully weighing the facts, that this correction is adequate; that the rates of mortality represent very nearly the mortality of the female population; and that the probabilities of female life, deducible from the mortality, are substantively true.

The rates of female mortality are in singular accordance with those deducible from observations on males; the mortality of females being slightly higher at the ages 10 to 35 than the mortality of males at home in England.—(Introduction to English Life Table, No. 3, pp. xiv–xxii.)

Constitution of a Life Table, or Normal Population.—The constituent individuals of a population are its elements; and the population is normal when its elements, arranged in corresponding groups, are in the same proportions as the elements of the Life Table. The births = deaths in the same time; to a given number born, the living at each year of age are in the same proportion as P_x to l_0; the rates of mortality are the same; the population lives a number of years after each age, represented by the calculated lifetime.

In a normal population there is an indissoluble connexion between (1) the numbers living, (2) the mean lifetime, (3) the births, (4) the deaths, (5) the rate of mortality, (6) the probable duration of life. Thus by the Life Table of Persons 1,000,000 annual births imply 1,000,000 annual deaths; sustaining a population of 40,858,184, of whom 20,426,138 are males, 20,432,046 are females; half of the persons living 45 years = the probable lifetime; and the mean lifetime being 40·858184 or nearly 41 years; that is = the mean age at death = the number of years of life falling to the share of the children born. To 41 persons living there is *one* birth, *one* death, annually; the rate of mortality is 1 in 41; and 41 is the mean duration of life.

It has been shown that the *rate of mortality* involves three elements,—time, numbers living, numbers dying; thus, if out of 102 living men of a given age 4 die at equal intervals in the year, 98 will live to the end of the year; so $\frac{98}{102}$ = the probability of living a year; $\frac{4}{102}$ = the probability of dying in the same time; and by hypothesis the 102 men in the year enjoy among them $\frac{102 + 98}{2} = 100$ years of life; now the years of life to be passed by the survivors in the next year will, if 4 die in the year, be 96, and thus the years of life will accumulate year by year, until the last life shall expire. All the years of life belong to the 102 men; and dividing the said years of life by 102 the mean afterlifetime is determined. Thus the units of the numbers that express living men, men dying, and years of life, are produced by men living a definite number of years and then dying.

By retaining one unit of time, and one living, in all cases, the variations of the numbers dying express the variations in the rate of mortality. By fixing the numbers living, and taking the death as a unit, the mean interval of time—which varies—between each death, will express the velocity of dying in the scale of time, under different conditions; and by making the living man a unit, the death becomes a unit, and the variations in the years of lifetime express the different degrees of longevity. By making the time a unit (one year), and the death a unit, the variations in the *numbers living*, out of which 1 death occurs annually—or the relative amount of resistance to death by life is expressed—under the given conditions. One death in *one* year to 41 *living* implies a mean lifetime of 41 years. It was shown before that 41 persons living through *one year* enjoy the same number of *years of life* as *one person* living *forty-one years*.

In a population which is disturbed by emigration, by immigration, by varying excesses of births over deaths or of deaths over births, or by pestilence, the *mean age of the dying* (G_0) can be determined from the registers by arranging the deaths consecutively in a column (d_x) at the various ages, and drawing up from this column the columns corresponding to l_x and L_x, or even to Q_x. But people are born in one place, die in another, and moreover the number of births is scarcely ever the same as the number of deaths. So there is no necessary connexion between the ages of these persons at death, the rate of mortality, the probability of living, or the mean duration of the lives of children born and living in precisely the same circumstances. The results nearly coincide sometimes with those deduced, on correct principles, from a life table; and the early life tables of Halley, Simpson, Dr. Price, and others, were constructed from the burial registers of Breslau, London, and Northampton, without any reference to the living. The errors of such tables are illustrated in the Appendix to the 8th Report of the Registrar General, where the old incorrect Northampton Table is compared with a new table for Northampton constructed on nearly the same plan as the English Table.*

The mean age of those who died in England in the 17 years 1838–54 was 29·4; whereas the mean lifetime of children born in England during the same period is 40·9 years by the life table. This reduction of the age at death, 11·5 years below the mean lifetime, is the result of the introduction of an excess of young lives; as in addition to the 380,631 births to balance the 380,631 deaths, 191,068, making 571,699 children in the whole were born annually and thrown into the population. The mean age of the dying = the mean age to which people live, in a normal population; but as our population is increasing, the mean age of the dying in a limited time is 11·5 years less than the mean lifetime. The mean age of the population of England was 26·4 years in 1851, instead of 32·1 years; so the excess of young people reduces the age of the nation by 5·7 years, or by half the difference $\left(=\frac{11\cdot 5}{2}\right)$ between the age at death (29·4) and the mean lifetime (40·9). Instead of living as long as they have lived (26·4 years), they will live about 35·6 years ($= E_{26\cdot 4}$.)—(Introduction to English Life Table, No. 3, pp. xxxi–xxxvii).

The Rate of Mortality and the probability of Dying.—If on an average of years out of 1,000 children born simultaneously, 149 die in the twelve months following the date of birth, the probability of dying is expressed by the fraction 0·149: that is the *death-chance* of a new-born infant under the given law of mortality. As 851 of them survive, 0·851 is the fraction to express the probability of living; it is the *life-chance.* Now ·851 + ·149 = 1 = life-chance + death-chance.

This probability is often expressed thus: the chances are 851 to 149 that a new-born child will live a year. The value of £1 payable if the child should live a year is 17*s.* (£·851); the value of £1 payable on the death of the child is 3*s.* (£·149); the chances in favour of life being greater than the chances in favour of death.

The lives may be looked at with a view to determine the persistency of the life-force; which is such in the present case, that 851 live out of 1,000 during one revolution of the earth; at the age of 20 it is such that 992 out of 1,000 men live a year. The proportions vary under varying conditions, but these variations do not accurately denote the vital force, which is only correctly measured on the *scale of mortality.*

The mortality is determined by the ratio which the deaths bear to the years of life. "The men living, and the time expressed in years, "multiplied into each other, produce the years of life with which the "deaths are compared. A year of life is the lifetime unit."† It is represented by one person living through a year; or by two persons living through half a year. A regiment of an average strength of 1,000 men during three years represents 3,000 years of life; and if the deaths in the three years are 60, the rate of mortality is thus expressed: $m = \frac{60}{3000} = \cdot 02$; or the mortality is said to be at the rate of 2 per cent.

* *See* Extract on pp. 480-2.

† *See* Introduction to English Life Table, pp. xiv–xx; and extract on pp. 485–8.

per annum. The 100 years of life are a fixed quantity; and as it is found that under various circumstances, and at different ages, the rate varies from 1 to 2, 3, 4, 5 up to 50, this scale serves to measure the life-force, or the complementary death-force, in the same way as the centigrade scale of the thermometer serves to measure heat.

A thermometer is not a convenient measure of heat unless at all temperatures it contains the same quantity of mercury, and unless each degree measures equal expansions of the mercury. If the mercury escapes, a correction is required to give the expansion of equal quantities of mercury at every degree of temperature. In observing with the barometer, the measure is adjusted at both ends, so as to give the exact height of the column above the mercury in its well.

So, to determine the rate of mortality on a strength of 1,000 men joined by no recruits, it is necessary to take their mean strength during the whole period of observation; for if one man dies at the end of a week, 999 only remain afterwards exposed to risk, and if the numbers are reduced at variable intervals to 990, to 985, to 911, to 700, to 600, and so on, it is evident that the years of life in the same time will be less than the years of life in a regiment which obtains a recruit for every casualty. All that is required in such cases is to take the observations so as to give the true years of life; and the ratio which these years of life bear to the deaths is the exact measure of the mortality. It is evident, on the other hand, that such a measure is not supplied by a comparison of the deaths in a year, for example, to the living at the beginning of that year. The results by this method are only strictly comparable when the deaths are in the same proportion and occur in the same periods of the year.

By the English Life Table 1,000 infants followed through their first year of age yield nearly 903 years of life; and the mortality is at the rate of $\frac{149}{903}$, or, more correctly, $\frac{149493}{902781} = \cdot 16559$. It is 16·559 per cent. per annum. The probability of dying is ·149493; and upon the erroneous assumption that this is the rate of mortality it would be 14·949 per cent. per annum; less by 1·610 than the true rate, with which it should never be confounded.

At other ages than the first year the rate of mortality serves to give the probability of living a year, and thus supplies the fundamental elements of a life table. The difference between the rate of mortality (m), and the probability of dying ($1-p$), becomes less in proportion as the two fractions diminish; for upon the hypothesis that the deaths in a year occur at equal intervals in the year, the relation of p and m is thus expressed:

$$p = \frac{1 - \frac{1}{2}m}{1 + \frac{1}{2}m} = \frac{2 - m.}{2 + m.}$$

(Supplement to 25th Annual Report, pp. iv–v.)

General Description of a Life Table; Healthy Districts.—The Transactions of the Royal Society contain the first life table. It was constructed by Halley, who discovered its remarkable properties, and illustrated some of its applications. The Breslau observations did not supply Halley with the data to frame an accurate table, for reasons which will be immediately apparent; but the conception is full of ingenuity, and the form is one of the great inventions which adorn the annals of the Royal Society.

Tables have since been made correctly representing the vitality of certain classes of the population; and the form has been extended so as to facilitate the solution of various questions.

In deducing the English Life Tables from the national returns, I have had occasion to try various methods of construction; and I now propose to describe briefly the nature of the life table, to lay down a simple method of construction, to describe an extension of its form, and to illustrate this by a new table representing the vitality of the healthiest part of the population of England.

The life table is an instrument of investigation; it may be called a *biometer*, for it gives the exact measure of the duration of life under given circumstances. Such a table has to be constructed for each district and for each profession, to determine their degrees of salubrity. To multiply these constructions, then, it is necessary to lay down rules, which, while they involve a minimum amount of arithmetical labour, will yield results as correct as can be obtained in the present state of our observations.

A life table represents a *generation of men* passing through *time;* and time under this aspect, dating from birth, is called age. In the first column of a life table *age* is expressed in *years*, commencing at 0 (birth), and proceeding to 100 or 110 years, the extreme limit of observed lifetime.

If we could trace a given number of children, say 100,000, from the date of birth, and write the numbers down that die in the first year, living therefore less than one year against 0 in the table, and on succeeding lines the numbers that die in the second, third, and every subsequent year of age until the whole generation had passed away, these numbers would form a *Table of Mortality*, showing at what ages 100,000 lives become extinct.

Again, if the 100,000 children were followed, and the numbers living on the first, on the second, and on every subsequent birthday until none was left, the column of numbers would constitute a *Table of Survivorship.* So if of 100,000 children born at a given point of time, the numbers dying (d_x) in each subsequent year were written in one column, and the numbers surviving (l_x) at the end of each year in another column, the two primary columns of the life table would be formed.

It is evident that if one of these columns is known the other may be immediately deduced from it; for if of 100,000 children born 10,295 die in the first year of age, 3,005 in the second year of age, it follows that the numbers living at the end of one year must be 89,705, at the end of two years 86,700. Upon adding the column (d_x) from the bottom up to the number against any age (x), the sum will represent the whole of the numbers *dying after that age;* and consequently the numbers *living at that age*, as shown in the collateral column (l_x).

The 100,000 children born at the same moment, and counted *annually* to determine the numbers *living* at *the end of every year*, would by our table completely pass away in less than 107 years. If another generation of 100,000, born a year afterwards, were followed, the numbers dying in the various years of age would not be very different, the circumstances remaining the same; and the numbers of those entering each year of age would vary inconsiderably from those of the first series. If 100,000 children again were born at annual intervals, and were subject to an invariable law of mortality, they would form a community of which the numbers living at each age would be represented by the successive numbers (l_x) in the life table. The sum of these numbers, by the new Table of Healthy Districts, would be 4,951,908. The births are here assumed to take place simultaneously at annual intervals; immediately before the births, therefore, in such a community its population would be 4,851,908, to which it would fall progressively from 4,951,908 by 100,000 successive deaths in the year.

The average number constantly living would be some number between 4,951,908 and 4,851,908; and it would be very nearly the mean of these limiting numbers.

In the ordinary course of nature, the births in a community take place in remittent succession; and if it is assumed that the 100,000 births occur at equal intervals over every year, it is evident that at any given date a certain number will be found living at all the intermediate points of age between 0 to 1 year, 1 to 2, 2 to 3, and all the remaining years of age. The population in the above instance would be found by enumeration to be nearly 4,899,665.

The annual *births* would be 100,000 in such a community. The annual deaths would also be 100,000; and by taking out the deaths at each year of age, from the parish registers of a single year, the second column (d_x) of the life table would be found. By adding this column of deaths up and entering the sum of the numbers year by year against every year of age (x), the third column (l_x) of the life table would be obtained; for it has been already shown that the numbers attaining any age x are equal to the numbers dying at that age, and all the subsequent ages. From the registers of the deaths, a table of the numbers of the *population living* in a parish *so constituted* could be immediately determined without any enumeration. Its deviations from the truth would be accidental; and they would be set right by taking the mean of many years. So also from a simultaneous enumeration of the *numbers living in each year of age*, the two columns d_x and l_x of the life table could be constructed without reference to any registry of the deaths at different ages.

The *mean age at death* in such a community would express the mean lifetime, or the expectation of life at birth; and the product of the number expressing the annual births multiplied into the mean age at death would give the numbers of the population.

The deaths in each year of age are called the *decrements of life.* The decrement in the first year is large; in the first five years the decrements of life are considerable; at the age of 10 to 15 they fall to their minimum; slowly increase to the age of 56; increase more rapidly until the maximum is attained at the age of 75; then decline gradually to 85, and after that more rapidly until every life is extinct at the age 107 by this table. ("On the Construction of Life Tables, illustrated by a new Life Table of the Healthy Districts of England," in the Transactions of the Royal Society, 1859, pp. 838–41.)

Basis and uses of the Healthy District Life Table.—Halley first pointed out the financial applications of the life table, and first calculated the values of life annuities. That branch of science, in the various forms of life insurance, has since received great developments. The new table shows that the duration of life, among large classes of the population by no means in unexceptionable sanitary conditions, exceeds the term of the ordinary tables, and proves that life annuities cannot be sold advantageously by offices, or by the Government, to large classes of lives for less than the values deducible from the new table.

A new branch of science has been developed since Halley's day,—it is the Science of Public Health. And here a new application of the life table is found.

It is probable, upon physiological grounds, that man goes through all the phases of his natural development in a hundred years; and that the period of active life seldom extends beyond eighty years. But this is a very indefinite measure, as the rates of mortality, in all the intermediate ages, are left undetermined after it has been ascertained in what proportions men attain the extreme limits.

Generations of men, under all circumstances, die at all ages; but the proportions vary indefinitely under different conditions from a slight tribute to death each year, down to the point of extermination by pestilence. If we ascertain at what rate a generation of men dies away under the least unfavourable existing circumstances, we obtain a standard by which the loss of life, under other circumstances, is measured; and this I have endeavoured to determine in the Life Table of Healthy English Districts. And recollecting that the science of public health was almost inaugurated in England by a former president of this society (Sir John Pringle), who encouraged and crowned the sanitary discoveries of Captain Cook, I feel assured that it will receive with favour this imperfect attempt to supply sanitary inquirers with a scientific instrument.

HEALTHY DISTRICTS.—Population, 1851. Deaths in the Five Years 1849 to 1853. Average Annual Mortality per cent.

Ages.	Population.			Deaths.			Average Annual Mortality to 100 living (m).		
	Persons.	Males.	Females.	Persons.	Males.	Females.	Persons.	Males.	Females.
1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
All ages -	996,773	493,525	503,248	87,345	43,736	43,609	1·753	1·772	1·733
Under 5 -	130,635	65,700	64,935	26,361	14,282	12,079	4·036	4·348	3·720
5— -	122,406	61,733	60,673	4,209	2,080	2,129	·688	·674	·702
10— -	110,412	56,651	53,761	2,377	1,087	1,290	·431	·384	·480
15— -	181,339	90,066	91,273	6,603	3,113	3,490	·728	·691	·765
25— -	136,892	65,422	71,470	5,869	2,675	3,194	·857	·818	·894
35— -	108,056	52,734	55,322	5,208	2,447	2,761	·964	·928	·998
45— -	85,244	42,383	42,861	5,252	2,698	2,554	1·232	1·273	1·192
55— -	62,857	31,105	31,752	7,001	3,568	3,433	2·228	2·294	2·162
65— -	39,453	18,860	20,593	10,313	5,173	5,140	5·228	5·486	4·992
75— -	16,737	7,718	9,019	10,297	4,946	5,351	12·304	12·817	11·866
85— -	2,614	1,097	1,517	3,581	1,555	2,026	27·399	28·350	26·711
95 & upwards.	128	56	72	274	112	162	42·813	40·000	45·000

The Healthy District Life Table was constructed in 1859 from the Census enumeration of 1851 and from mortality observations extending over the five years 1849 to 1853 in 63 districts of England and Wales which showed during the ten years 1841–50 a mean annual death-rate not exceeding 17 per 1,000 persons living. It has been found by experience that this Healthy District Life Table expresses very accurately the actual duration of life among the clergy and other classes of the community living under favourable circumstances.—("On the "Construction of Life Tables, illustrated by a new Life Table of the "Healthy Districts of England," in the Transactions of the Royal Society, 1859, pp. 838–41.)

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